Repeated Interactions Under Costly Enforcement.*

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Abstract

We study repeated irreversible investment. We assume that *ex ante* ownership rights are incomplete and *ex post* property allocation is endogenous. In a stage game, principal can renege on *ex ante* contract with agents (investors). To capture that ownership rights depend on prior arrangements, we introduce a dynamic game, in which player *ex ante* ownership shares are equal to their *ex post* shares in the stage game of the previous period. With the commitment constrained principal, equilibrium of the dynamic game features cyclical changes in investment. These cycles indicate that contractual incompleteness alone causes output fluctuations. Thus, when costly contracts result in constrained commitment, persistent cycles in output occur, resembling business cycles.

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JEL Classification Codes: C73, E30

1 Introduction

> When taxes are too high, people go hungry
> When the government is too intrusive people lose their spirit
> Act for the people's benefit. Trust them, leave them alone
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> Lao Tzu, *Tao Teh Ching*
> Book II, Section 57, (appr. 500 BC)

It is costly and sometimes impossible to enforce contracts. In this paper, we study repeated irreversible investment. In a stage game, player ownership rights are based on an *ex ante* contract, which the principal can revise *ex post* at an exogenous

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cost, continuous and concave in the magnitude of his ownership share adjustment (increase). In our setting, *ex ante* ownership rights are incomplete and allocated endogenously. We introduce infinitely repeated dynamic game, in which stage games in subsequent periods are interdependent. Starting from the second period, player *ex ante* ownership shares are equal to their *ex post* shares in the stage game of the previous period. When principal is commitment constrained, equilibrium of the dynamic game features persistently repeated cyclical changes in investment and ownership allocations, which indicate that contractual incompleteness alone causes output fluctuations, i.e., business cycles. The cyclical changes in production (investment) and ownership (surplus sharing) allocations are the most interesting features of our model.

Mathematically, costly contracts and imperfect property rights are identical. Imperfect property rights can always be modelled as costly contracting, with non-contractible cases treated as infinitely costly.[1] In such settings, the classical Coasian efficiency does not hold as the Coase (1960) theorem does not apply to costly contracting. The literature identifies several causes of inefficiencies. The contract theory literature deals with an important subset of such inefficiencies, known as a hold-up problem (also called principal-agent problem).[2] In this literature, the most frequent cause of inefficiencies is the difference between player *ex ante* and *ex post* incentives and bargaining strengths. This reflects empirical evidence (see Williamson (1975), (1985) and North (1990)) of frequent wedge between *ex ante* and *ex post* incentives, because relative *ex ante* and *ex post* bargaining strengths of the players differ.

This literature focuses on alleviating the suboptimality of investment allocation, when regulations, informational constraints or, in general, non-zero transaction costs, violate Coase (1960) result.[3] Exogenous distribution of the contact surplus

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[1]Costly contracts imply imperfect or incomplete property rights for an object of the contract. When property allocation specified by an *ex ante* contract can be modified *ex post* at a cost by the player(s), property rights are, clearly, incomplete.

[2]See Holmstrom and Tirole (1989) review, and Tirole (1999) for a summary of achievements and unresolved questions of this research area.

[3]The literature bearing on the principal-agent problem is far too extensive for reviewing, or even listing it here. For recent developments see *Review of Economic Studies,* (1999), Vol. 66, Issue 1. Some references to the literature will be found throughout the paper though we make no claim to completeness.

is a standard assumption of this literature. Full surplus is allocated to one of the players, or the players get equal surplus shares (Nash bargaining), or some other ad hoc restriction is imposed on player surplus shares. The resulting equilibrium and player payoffs depend on environment. In this sense, ownership allocation is endogenously determined. Nevertheless, the assumption of exogenous surplus distribution leads to allocations, which differ from the ones that emerge if surplus distribution can be chosen by the players. Fixing surplus shares exogenously is equivalent to imposing an exogenous constraint. Such a constraint could preclude the players from resolving the hold-up thought an endogenous choice of surplus distribution. Our focus is a typical hold-up caused by the wedge between player *ex ante* and *ex post* incentives. We suggest that it is harder to study the determinants of such a wedge and its effect on investment incentives, when player surplus sharing rule is fixed exogenously. Our model is designed to study the determinants of the wedge between *ex ante* and *ex post* incentives, and its effect on investment incentives. We allow the principal to choose *ex ante* surplus division, and, reflective of contractual imperfections, permit him to adjust this surplus division *ex post* at an exogenous cost increasing in the size of required adjustment of his share.

The game theory literature, which addresses a division of a unit size asset is complementary to agency literature. This literature focuses on the factors that affect player surplus shares.[4] We combine these approaches to investigate the division of a variable size asset, whose size is endogenously determined by the surplus distribution between the players, who interact repeatedly and choose the surplus sharing rule in each period.

First, we introduce a stage game, borrowed from Schwartz (2000), in which the asset is divided only once. We show that the game has a unique equilibrium, in which the principal incurs positive *ex post* expenses to increase his *ex ante* ownership share of the asset. We interpret these expenses as spending on contract reneging (which includes bureaucratic and/or legal fees, or even penalties for the breach of the *ex ante* contract). We call an equilibrium with a positive spending on reneging the *reneging* equilibrium. We compare the stage game to the one in which the players are committed, i.e., the *ex ante* contract cannot be modified *ex post*. Then, *ex ante* and

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[4]See for example, Rubinstein (1982).

ex post surplus shares (and, thus, ownership allocations) are identical, and ownership rights are well defined and secure. We prove the existence and the uniqueness of equilibrium outcome in this game, and call it the *commitment* outcome. In the *reneging* equilibrium of the stage game, investment and player payoffs are lower than in the *commitment* outcome.

Anderlini and Felli (1997) study property rights by using bargaining games. They consider a hold up problem in the presence of *ex ante* contract costs and investigate the conditions under which socially efficient contracts are infeasible. In their initial setup, the distribution of bargaining power across agents is exogenous, and the resulting contracts are constrained inefficient. The inefficiency arises for a certain range of the bargaining powers of the players. Further, Anderlini and Felli (1997) endogenize the distribution of the surplus across players. For a certain range of *ex ante* contract costs, socially desirable contracts are not feasible, irrespective of the surplus distribution. Anderlini and Felli (1997) suggest that when the potential surplus depends on its distribution, the hold up problem is less acute.

Our model is analogous to the Anderlini and Felli (1997) setup when the potential surplus is dependent on its distribution. While in their paper only two-party games are considered, we consider multi-party contracts, and our model permits multi-period contacts, which we study below. Their model is applicable to a wider range of environments, while our focus is the mechanism behind surplus division. They use different modeling techniques, and focus on normative issues and *ex ante* inefficiencies. Our approach is more applied: we focus on inefficiencies stemming from the divergence player *ex ante* and *ex post* incentives.

Next, we consider two infinitely repeated games. The first game is a supergame, with the long-lived principal and short-lived investors, based on our stage game. The second game, which we call *dynamic*, is identical to the first, except the stage games played in subsequent periods are interrelated. Starting from the second period, player *ex ante* ownership shares are equal to their *ex post* shares in the stage game of the previous period. The condition captures that ownership rights depend on prior arrangements.

In some cases, the first game adequately reflects reality, in others, the *dynamic* game is more accurate. For example, to model the effects of taxation on investment,

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consider the game between investors and government (as the principal), where government share is a tax rate. The condition imposed in the dynamic game means that the realized, that is *ex post* 2002 tax rate is equal to the *ex ante* 2003 tax rate. In other words, government cannot set a "special" *ex ante* tax rate for 2003 tax year. The second game suggests stronger enforcement institutions (legal, cultural, etc.) constraining the players. Strong institutions have more mechanisms to mitigate government commitment conflict than the weak ones, which exacerbate government credibility problem due to fewer options to alleviate the commitment deficiency.

In each repeated game, we prove the existence of equilibrium, and study its properties. We call the lowest discount factor at which each player receives at least his commitment payoff in some equilibrium, the *commitment* discount factor. We prove that for discount factors less than *commitment* discount factor, each game has a unique equilibrium. Further, we show that this equilibrium is a *reneging* equilibrium, with investment and player payoffs lower than in the *commitment* outcome but higher than in the equilibrium of the stage game.

In the equilibrium of the second game, the principal's ownership share is adjusted upward for a finite number of periods (denoted a T-cycle), then drops to its initial level, where a new cycle of upward adjustments starts. Thus, when the principal is commitment constrained, an equilibrium of the *dynamic* game has an interesting feature of cyclical changes in investment, resembling business cycles. Such a pattern of tax or/and tariff behavior is frequent. In our view, this pattern is caused by government commitment limitations.

Business cycles, that is recurrent cyclical movements of output, are empirically important in present-day economy. The search for reasons behind business cycles receives considerable attention in the macroeconomic literature. To generate business cycles, a number of papers explores the possibility that frictions of financial markets may amplify and propagate shocks to the economy.[5] Kocherlakota (2000) synthesizes the ideas of Kocherlakota (1996), Cooley, Marimon, and Quadrini (2000), and Kiyotaki (1998). He demonstrates that endogeneity of credit constraints is crucial

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[5]Bernanke et. al. (1998) generalize this literature. They construct a financial accelerator, and show how endogenous developments in credit markets work to amplify and propagate shocks to the economy. They conclude that with reasonable parameterizations of the model such endogenous developments have significant influence on business cycle dynamics.

for generating empirically plausible amplification of income shocks with realistic parameters.

In our model, the cycles are driven by imperfect commitment, which leads to time-inconsistent government policies. Several existing studies, extending Kydland and Prescott (1977), point out commitment deficiencies as a cause of business cycles. For example, Phelan (2001) develops a model of government reputation, which combines trigger models (where good equilibria are supported by a threat of getting into a bad one) and reputation models (where the type of government is imperfectly observable, which creates incentives for bad types to pretend being good). Phelan (2001) assumes that government's type (good or bad) changes following a Markovian process. With such opportunistic governments, the equilibrium features gradual rebuilding (N-periods) of trust to government after expropriation occurs. His N-periods resemble our T-cycles.

The connection of commitment and business cycles is not a new idea. It has been considered by the literature that examines the patterns of risk sharing in infinite-horizon environments with exogenous random incomes and costly commitment.[6] Kocherlakota (1996) allows the allocation of time between labor and leisure to be endogenous. He studies the environment in which the outside enforcement of risk-sharing is infinitely costly. He demonstrates that when preferences over consumption and leisure are nonhomothetic, and individuals face idiosyncratic shocks to labor productivity, efficient allocation of risk results in persistent cycles in aggregate output. Another mechanism to generate cycles of aggregate output is through market imperfections, which were shown to lead to cycles (for example, Kiyotaki and Moore (1993) consider imperfections in borrowing against the human capital).

Although the equilibrium of the *dynamic* game features persistent output cycles, the mechanism behind them differs from the existing literature. In the *dynamic* game cycles could be present in the absence of uncertainty. They persist even with perfectly competitive capital markets.

The aforementioned macroeconomic literature relies on the presence of uncertainty or market imperfections, or both, to generate the output cycles. Neither assumption is necessary to get cycles in the equilibrium of our model. The key

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[6]See also Scheinkman and Weiss (1986), Thomas and Worrall (1990), and Kocherlakota (1994).

factors that generate equilibrium cycles in the *dynamic* game are investment irreversibility, and imperfect (i.e. costly) property rights, dependent on prior ownership arrangements. This dependence is crucial for generating output cycles. In case of our supergame with investment irreversibility and commitment imperfections, but surplus division independent of previous periods, equilibrium cycles of output do not occur. Perhaps, our model compliments traditional explanations of business cycles. A combination of our game with the business cycles literature would make it possible to measure the relative importance of the government commitment problem and other causes of output volatility.

To summarize, we propose an infinitely repeated game with a new twist: player actions in subsequent periods are interdependent. This game permits to analyze dynamic environments with ownership allocation dependant on prior arrangements. Equilibrium cycles indicate that government credibility problem alone causes output fluctuations, i.e., business cycles. Also, we contribute to the literature on repeated costly contracting. We emphasize that costly contracts make production and property allocations interdependent. We propose a model with jointly determined ownership and production allocations.

The paper is organized as follows. In Section 2, the stage game is presented and its equilibrium properties outlined. In Sections 3, 4 and 5 repeated games are introduced, and their equilibria analyzed, with comparative equilibrium analysis provided in Section 6. The discussion and conclusion are presented in Section 7. Proofs and technical details are relegated to Appendices.

2   Stage Game

Let Γ denote a complete information game of $N+1$ players, a principal and N agents (investors), and the following order of moves. First, the principal chooses his *ex ante* asset share $x \in [0, 1]$. Then, the investors simultaneously and independently choose their irreversible investments into the asset $q_n \in [0, \infty)$. The aggregate investment

$$Q = \sum_{n=1}^{N} q_n$$

determines the asset value $P(Q)$. Third, the principal can adjust his *ex ante* share to an *ex post* value $y \in [0,1]$ at an exogenous cost.

The game Γ models endogenous allocation of ownership in environments with contractual incompleteness. The n-th investor objective is to maximize his profit, $\Pi_n(x, y, \mathbf{q})$, which is equal to the value of his *ex post* asset share net of his opportunity cost of funds iq_n, with i denoting an outside option return:

$$\Pi_n(x, y, \mathbf{q}) = \frac{q_n}{Q}(1 - y)P(Q) - iq_n : n = 1, \dots, N, \tag{1}$$

where $\mathbf{q} = (q_1, ..., q_N)$ is the vector of investments. The principal's objective is to maximize his net surplus, $V(x, y, \mathbf{q})$, which is equal to the value of his *ex post* ownership share net of his adjustment (reneging) cost $B(y - x)$:

$$V(x, y, \mathbf{q}) = yP(Q) - B(y - x) \tag{2}$$

The asset value is continuous, concave and three times continuously differentiable for $Q \in (0, \infty)$. In the absence of commitment conflict, investment in the asset is positive[7]:

$$P'(Q) > 0, \ P''(Q) < 0, \ \lim_{Q \to 0} P'(Q) > i.$$

The function B is continuous, convex, three times continuously differentiable for $z \in (0,1)$ and equal to zero for $z \leq 0$:

$$B'(z) > 0, \ B''(z) > 0 \quad \text{for} \quad \forall z \geq 0, \quad \text{and} \quad B(z) \equiv 0 \quad \text{for} \quad \forall z \leq 0,$$

which implies that a *reduction* of the *ex post* ownership share is free for the principal. To simplify the exposition, we assume:

$$P'''(Q) \leq 0, \ B'''(Q) \geq 0 \quad \text{and} \quad \lim_{z \to 0} B(z) = 0, \ \lim_{z \to 0} B'(z) = 0, \tag{3}$$

which entails a zero fixed cost. The equilibrium concept used to analyze the game

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[7]Which means that the asset value evaluated at zero exceeds the outside option return.

Γ is a subgame perfect Nash equilibrium symmetric with respect to the investors.

Definition 1 *A reneging equilibrium is an equilibrium in which the principal's ex post expenses are positive.*

Let $\hat{\Gamma}$ denote the game Γ, in which *ex ante* and *ex post* contracts are identical.

Theorem 1 *There exists a unique equilibrium in each of the games Γ and $\hat{\Gamma}$. The equilibrium of the game Γ is a reneging equilibrium, with investment below, and the principal's ownership share above the respective values in the game $\hat{\Gamma}$.*

Proof. See Appendix. ∎

In the game $\hat{\Gamma}$ the principal is committed to an *ex ante* contract, and from Theorem 1 the equilibrium of the game $\hat{\Gamma}$ is unique, permitting the following definition:

Definition 2 *Let $\hat{\Gamma}$ denote the game Γ in which $x \equiv y$. Equilibrium outcome of the game $\hat{\Gamma}$ is called the commitment outcome.*

3   The Game G

Let G denote δ-discounted infinitely repeated game, with $\delta \in (0, 1)$, between the long-lived principal and short-lived investors, with the game Γ as a stage game. In the game G the principal maximizes his discounted payoff and each investor – his per period profit.

The game G models environments, in which players cannot (or have no incentives) to interact (play) with each other throughout the entire game. For example, such a game plausibly depicts interactions of government (principal) and the telecommunications industry (investors). The principal's inability to commit in the game Γ makes it likely that the game G would be between the long-lived principal and the short-lived investors. Moreover, we expect the assumption of short-lived investors to become even more appropriate in the future, due to a trend of financial liberalization and globalization of financial markets. To summarize, the game G has the following elements:

$$G = G(P, B, N, i, \delta),$$

which are fixed, unless the reverse is stated explicitly. The equilibrium concept for the game G is a subgame perfect Nash equilibrium, symmetric with respect to investors, which cannot be Pareto dominated. Definitions 2 and 1 are applicable for our infinitely repeated games.

Definition 3 *Let $\hat{\delta}$ denote the lowest discount factor at which each player receives his commitment payoff in some equilibrium. We call $\hat{\delta}$ the commitment discount factor.*

Theorem 2 *An equilibrium of the game G exists. For $\delta \in (0, \hat{\delta})$ [the principal's preferred] equilibrium is unique, and is a stationary reneging equilibrium.*[8]

Proof. See Appendix. ∎

Our proof of Theorem 2 has five steps. First, we notice that the equilibrium existence follows from the continuity and compactness of player action spaces, and quasi-convexity of their payoffs. Second, we prove the uniqueness of each player stationary *preferred* equilibrium, where under "preferred" we mean the one in which the player's payoff is his maximum equilibrium payoff. Third, we show that for $\delta \in (0, \hat{\delta})$ the principal's and investor stationary preferred equilibria (PPE and IPE) coincide, which provides the uniqueness of stationary equilibrium. Fourth, we prove that for $\delta \in (0, \hat{\delta})$ the PPE is a reneging equilibrium. Lastly, we assume the existence of a non-stationary equilibrium and show that in this case, multiple stationary equilibria would exist, which contradicts the uniqueness of stationary equilibrium.

The equilibrium uniqueness for $\delta \in (0, \hat{\delta})$ is driven by the principal's commitment deficiency. The reneging equilibrium is a balance of two effects: principal's higher reneging expenses (thus, surplus loss) and gross payoff increase (thus, potential surplus gain), which accompany the asset value increase. The equilibrium uniqueness for $\delta \in (0, \hat{\delta})$ reflects that player objectives are perfectly aligned: each player payoff increases with δ.

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[8]Theorem 2 has a stronger version, than the proof in Appendix below. We can show that: *An equilibrium of the game G exists. For $\delta \in (0, \hat{\delta})$ the equilibrium is unique, and is a stationary reneging equilibrium.* When $N = \infty$ the stronger version of Theorem 2 follows from our proof in Appendix immediately.

When $\delta > \hat{\delta}$, player interests no longer coincide. When equilibrium surplus exceeds the *commitment* outcome surplus, some players experience an absolute payoff decrease along with a reduction of their surplus share relative to the *commitment* outcome values.

Proposition 1 *When $\delta \in (0, \hat{\delta})$, in the equilibrium of the game G, the principal's reneging expenses and ex post share decrease with δ and investment increases with δ .*

Proof. See Appendix. ∎

From Theorem 2, when $\delta \in (0, \hat{\delta})$, the equilibrium of the game G is unique, which permits us to prove Proposition 1 through explicit derivation of player best response functions.

From Proposition 1, for $\delta \in (0, \hat{\delta})$ in the equilibrium of the game G the asset value and player surplus increase with δ. The principal's equilibrium payoff increases with δ, despite the fact that his surplus share of the asset decreases with δ. Our results rationalize the empirical observation of high fluctations of capital taxes in countries with low government credibility and weak institutional arrangements[9]. From Proposition 1, commitment constrained governments renege on their *ex ante* tax promises. Thus, the actual (*ex post)* tax exceeds the promised one. Such instability of economic policies are characteristic for countries with weak legal institutions. From Proposition 1, as δ converges to $\hat{\delta}$ equilibrium reneging expenses converge to zero, and the principal's share converges to his commitment outcome share, hence:

Corollary 1 *For $\delta = \hat{\delta}$ the equilibrium of the game G is unique, and it is the commitment outcome.*

From Proposition 1 and Corollary 1, if $\delta \in (0, \hat{\delta})$, player equilibrium payoffs are lower than their *commitment* outcome payoffs.

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[9]In countries with a strong legal system, low government credibility can be mitigated by legal mechanisms. Government credibility problems are exacerbated by weak institutions due to fewer options that such institutions have to alleviate commitment deficiencies.

4   The Game Gl

So far, we have assumed that the players do not necessarily contract with each other throughout the entire game. Nevertheless, sometimes players have no choice but to deal with each other repeatedly. Consider, for example, an economy with no trade or capital mobility, and model capital taxation as the game G. Then, the government is the principal, the tax is his share, and GDP – the asset value. Here the players are committed to play with each other for the entire game: they have no other options. In this case, the game with all players long-lived is an appropriate model.

Let Gl denote δ-discounted infinitely repeated game, with a stage game Γ and long-lived players. In the game Gl each player maximizes his discounted payoff. The equilibrium concept for the game Gl is the same as for G.

Theorem 3 *An equilibrium of the game Gl exists. For $\delta \in (0, \hat{\delta}_{Gl})$ [principal's preferred] equilibrium of the game Gl is unique, and is a stationary reneging equilibrium.*

   Proof. See Appendix.   ∎

In the game Gl investor ability to punish the principal for deviation is stronger than in the game G. In the game G, Nash reversion is the strictest punishment that can be imposed on the principal; in the game Gl the investors can employ a zero investment.

Besides having better means to punish the principal's deviations, in the game Gl investors could be strategic: they do not necessarily maximize per period profit in each period. But from Theorem 3, for $\delta \in (0, \hat{\delta}_{Gl})$, the investors do maximize their per period profit in every period, exactly as in the game G. The *reneging* equilibrium in the game Gl resembles the one in G:

Proposition 2 *The commitment discount factor in the game G is higher than in Gl. For $\delta \in (0, \hat{\delta}_{Gl})$ in the equilibrium of the game Gl investment is higher, and the principal's reneging expenses and ex post share lower than in G.*

   Proof. See Appendix.   ∎

Corollary 2 *For $\delta \in (0, \hat{\delta}_{Gl})$ the game Gl Pareto dominates G.*

Proof. Follows from Proposition 2 ∎

We expect that the players would employ available, and, perhaps, create special mechanisms to turn the game G into Gl. We suggest that financial market restrictions are so persistent because they serve this purpose. Clearly, when investment market restrictions are relaxed, the game Gl turns into G, which results in inferior equilibrium allocation, with lower investment and player payoffs. This inference agrees with the literature that examines effects of financial liberalizations[10].

5   The Dynamic Game

Let D denote the game G in which the principal's *ex post* action is identical to his *ex ante* action in the subsequent period:

$$x^{t+1} = y^t \text{ for } \forall t > 1, \tag{4}$$

where the superscript t refers to the stage game played in period t. For example, in the game between investors and government, where government share is a tax rate, equation (4) states that the realized, i.e., the *ex post* 2002 tax rate is equal to the *ex ante* 2003 tax rate, which means that the government cannot set a "special" *ex ante* 2003 tax rate.

Equation (4) is an additional constraint on the principal, which we interpret it as an indication of the existence of stronger mechanisms to constrain the principal from reneging. This implies that the game D fits for modelling more advanced institutional environments than the ones for which the game G is appropriate. We

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[10]Demirgüç-Kunt and Detragiache (2001) review the empirical data linking financial development and economic growth. They investigate whether a relationship between the banking crises and liberalization is stronger in the countries with weaker institutions. The study provides a variety of robustness checks, and concludes:

> "In the countries where the rule of law is weak, corruption is widespread, the bureaucracy is inefficient, and contract enforcement mechanisms are ineffective, financial liberalization tend to have a particularly large impact on the probability of the banking crises."

attribute constraint (4) to improved contractual institutions compared to the ones where the game G is applicable. The equilibrium concept for the game D is the same as for G.

Theorem 4 *An equilibrium of the game D exists; for $\delta \in (0, \hat{\delta}_D)$ it is unique, and is a reneging equilibrium.*

Proof. See Appendix. ■

The existence of an equilibrium in the game D follows from the continuity and compactness of player action spaces, and quasi-convexity of player payoffs. To prove that for $\delta \in (0, \hat{\delta}_D)$ the equilibrium is unique, we introduce the concept of a *T-cycle*.

Definition 4 *We call a T-period sequence starting in period τ a T-cycle, if:*

$$y^t < y^{t+1} \quad for \ \ t = \tau, \dots, \tau + T - 1 \quad and \quad y^{t-1} > y^\tau = y^{\tau + T + 1}.$$

Along the T-cycle, the principal's ownership share increases with t and falls in period $\tau + T + 1$ to his share in period τ. Definition 4 implies that any T-*cycle* has a finite length.

Proof of equilibrium uniqueness for $\delta \in (0, \hat{\delta}_D)$, is summarized in five steps. First, we notice that for $\delta \in (0, \hat{\delta}_G)$ the principal's maximum sustainable non-*reneging* payoff in the game G is lower than his equilibrium payoff. Second, we show that the principal's minmax payoff in the game D is bounded by his minmax payoffs in the games G and Gl. Third, we prove that there exists $\delta > 0$, at which the game D has a *reneging* equilibrium (Main Lemma). As δ approaches zero, the principal's maximum achievable non-*reneging* payoff converges to zero in the games G and Gl, and in D as well (from Step 2). But his equilibrium payoffs in the games G and Gl do not converge to each other with δ converging to zero. We show that there exists a $\delta > 0$, for which the principal's maximum *reneging* payoff is higher in the game D than G, and does not converge to his equilibrium payoff in the game G as δ converges to zero, from this Main Lemma follows. Fourth, we extend Main Lemma to $\delta \in (0, \hat{\delta}_D)$. Fifth, we notice that the arguments that provide the equilibrium uniqueness in the games Gl and G are applicable in the game D. Thus, the equilibrium of the game D is unique for $\delta \in (0, \hat{\delta}_D)$.

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From Theorem 4, for $\delta \in (0, \hat{\delta}_D)$, the game D has a unique reneging equilibrium; from this the existence of a unique equilibrium T-cycle follows:

Proposition 3 *For $\delta \in (0, \hat{\delta}_D)$ equilibrium outcome of the game D consists of the repetition of a unique T-cycle. The size of the principal's share adjustment and the asset value decrease in t for $t > 1$, where $t = 1, ...T$.*

Proof. See Appendix. ■

Proposition 3 connects the pattern of taxation with government commitment capacity. Theorem 4 and Proposition 3 suggest that imperfectly committed government exhibits a 'circular' pattern of taxation. Tax is adjusted upward for a finite number of periods, and, thereafter, drops to its initial level. Then, a new cycle of upward adjustments starts. Clearly, taxes affect investment: circular tax changes induce counter-cyclical changes in investment. Thus, from Proposition 3 commitment imperfections interplay with business cycles. We suggest that the pattern of business cycles as affected by legal and enforcement institutions, because these institutions affect government commitment capacity.

Tax incentives are widely used to remedy recessions as they are proven to help. Interestingly, the results of Proposition 3 give a new explanation of why a favorable tax regime helps to curb recession.

Our model can be tested. The results of this section imply that the responsiveness (i.e., the duration and volatility of business cycles) to tax incentives, and the magnitude of appropriate tax adjustments are indicative of government commitment imperfections. Higher magnitude of tax adjustments reflects more severe government commitment imperfections.

Corollary 3 *For $\delta \in (0, \hat{\delta}_D)$, equilibrium profit, in the game D, is strictly positive even when investment market is perfectly competitive.*

Proof. See Appendix. ■

Interestingly, from Corollary 3 when $\delta \in (0, \hat{\delta}_D)$ and investment market competition is high, equilibrium profits in the game D are higher than investor profits, that follow from technology and market competition characteristics in the absence

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of the T-*cycle* equilibrium. We suggest that Corollary 3 provides an explanation of high corporate profits on investment in developing economies (after controlling for risk). Our result rationalizes "popular wisdom", which regards investments in the countries with relatively weak institutions as a lucrative opportunity.

6 Comparative Analysis

The game D with long-lived investors is unrealistic: it is difficult to devise a mechanism, which would make the player (investor) stay in such a game credible, especially given the current trend of globalization. Thus, we compare the equilibria of the games G, Gl and D only.

Theorem 5 *The commitment discount factor in the game D is bounded by the ones in Gl and G. For $\delta \in (0, \hat{\delta}_D)$ player equilibrium payoffs are higher in the game D than G. For $\delta \in (0, \hat{\delta}_{Gl})$ principal's equilibrium payoff is higher in the PPE of the game Gl than D.*

Proof. See Appendix. ■

From Theorem 5, for $\delta \in (0, \hat{\delta})$ the principal's equilibrium payoff is higher in the game Gl than in D. Clearly, the game Gl permits stronger means for punishing the principal's deviation than the game D. Interestingly, from Theorem 5 and Corollary 3 when investment market competition is sufficiently high (close to perfectly competitive) investor equilibrium profits are higher in the game D than Gl. Thus, high investment market competition creates conditions for institutional developments as such developments benefit investors.

Corollary 4 *Equilibrium in each of the games G, Gl and D is unique for its commitment discount factor, with commitment outcome being an equilibrium outcome in each game at its $\hat{\delta}$.*

Proof. Follows from above. ■

From Corollary 4 and Theorems 3 — 5 in each repeated game, its *commitment* discount factor is the lowest discount factor at which *commitment* outcome sus-

tainable, the lowest discount factor with non-reneging equilibrium; and the highest discount factor at which the equilibrium is unique.

7 Discussion and Conclusion

In many environments, *ex post* contract violation is routine. The wedge between *ex ante* and *ex post* incentives has numerous reasons (technological, informational, regulatory, etc.), but it leads to a common consequence: impossibility to sustain Pareto frontier outcomes in equilibrium. We have shown that when due to investment irreversibility the equilibrium *ex post* contract diverges from the *ex ante* one, Pareto frontier outcomes are not sustainable, and ownership and investment allocations are interdependent. Thus, our contribution to the modelling of repeated costly contracting is a new approach, in which production and ownership allocations (investment and ownership shares) are endogenous. The cyclical changes of output and player surplus distribution appear to be the most interesting features of our dynamic game, making it attractive as a tool for modelling repeated contractual interactions under costly contracting.

We leave causes of irreversibility unspecified to permit a wide range of applications: from sunk cost or relationship-specific investment irreversibility to the ones generated by the regulator's commitment conflict. Clearly, our game can be extended to analyze agency conflict in the environments with symmetric uncertainty, for example, when investment return is subject to random shocks.

Our games can be used to evaluate and develop legal rules, such as optimization of the structure of penalties for a breach of contract. Our model can be extended to the games, with non-zero fixed costs or non-zero principal's costs of his ownership share reduction. Schwartz (2002) considers non-repeated *reneging* game: a generalization of our stage game, in which which all players, can affect *ex post* ownership allocation, as the principal can do in the game Γ. The *reneging* game models "the two sided reneging". Infinitely repeated games based on the *reneging* game, can be used to model incentives for specific investment, in environments with repeated interactions and costly contracting, see Schwartz (2002).

Our model has testable implications. For example, from Proposition 1 more

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commitment constrained governments should have higher magnitudes of tax adjustment, and therefore, larger output volatility. Cooley, Marimon, and Quadrini (2000) reach a similar conclusion. They argue that "the lower is degree of contract enforceability, the larger is the macroeconomic instability," and support their statement with and theoretical model and empirical evidence.[11]

We focus on discount factors below the *commitment* one (at which the commitment problem is resolved). For higher discount factors our results are about surplus redistribution rather than commitment imperfections. Still, the question is related to the question of the determinants of ownership allocation. We hope that our model is useful for further research of these issues.

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Appendix

Proof of Theorem 1. The proof is borrowed from Schwartz (2000). A sketch of the proof is presented below.

In symmetric equilibria, investor actions are identical and, since the principal's objective depends on aggregate (not on the individual investors) investment, it is sufficient for the principal to condition his actions on aggregate investment.

To simplify, we say that a function is defined on a closed interval, when the function is actually well defined only on the respective open interval. At the boundary points we consider the left or the right limit of the function. Consider the following system of equations:

$$P(Q) - B'(y - x) = 0, \tag{5}$$

$$(1 - y)A(Q) - i = 0, \tag{6}$$

where $Q \in [0, \infty)$, $x, y \in [0, 1]$,

$$A(Q) = \frac{1}{N}P'(Q) + (1 - \frac{1}{N})\frac{P(Q)}{Q}. \tag{7}$$

From properties of the function P, the function A is continuous and twice continuously differentiable for $Q \in [0, \infty)$ (the function A is a weighted average of marginal and average costs).

From Schwartz (2000) [proof of Theorem 1] and equation (3) the system of equations *(5) - (6) has a unique solution* $(y(x), Q(x))$ for any $x \in [0, 1]$, and it is *continuous, twice continuously differentiable,* and $Q'(x) < 0$:

$$Q'(x) = \frac{i}{(1-y)^2 A'(Q) - \frac{iP'(Q)}{B''(y-x)}} < 0, \tag{8}$$

where

$$A'(Q) = \frac{1}{N} P''(Q) + (1 - \frac{1}{N}) \frac{1}{Q} \left[P'(Q) - \frac{P(Q)}{Q} \right] < 0. \tag{9}$$

Equations 8 and 9 are negative from the properties of the functions P and B. Due to equation (3) for any $x \in [0, 1)$, $y(x) > x$, because from equation (5) for any $Q > 0$ $y(x) > x$, and from equation *(6),* for any $x \in [0, 1)$, $Q(x) > 0$. Thus, there exists a unique equilibrium in the subgame of the game Γ which starts at any $x \in [0, 1]$, and unique best responses $Q^*(x)$ and $y^*(x) = y(x, Q^*(x))$.

[[Notice that the imposition of equation (3) simplifies the equilibrium structure of the game Γ, and makes the proof easier. Equation (3) permits us to eliminate the discontinuities in the best response schedules $Q^*(x)$ and $y^*(x)$.]]

The Game with Complete Commitment

Proof. Let $\hat{\Gamma}$ denote the game Γ, in which the principal's *ex post* action is restricted to be equal to his *ex ante* action. From Schwartz (2000), the game $\hat{\Gamma}$ has a unique Pareto-dominant equilibrium, which we call the commitment outcome. For any $x \in [0, 1]$, in the game $\hat{\Gamma}$ there exists a unique aggregate best response $\hat{Q}(x) = N \times \hat{q}(x)$, where $\hat{q}(x)$ is each investor's best response. The functions $\hat{Q}$ and $\hat{q}$ are continuous, twice continuously differentiable and decreasing in x:

$$\hat{q}'(x) < 0, \quad \hat{Q}'(x) < 0 : \forall \ x \in (0, 1)$$

From Schwartz (2000) [Proof of Theorem 1] $\hat{Q}(x)$ is a solution of equation

$$(1 - x)A(Q) - i = 0,$$

and $\hat{Q}'(x)$ is equal to:

$$\hat{Q}'(x) = \frac{i}{(1 - x)^2 A'(\hat{Q}(x))} < 0, \tag{10}$$

and $\hat{x} = \min_{x_i \in \hat{X}} x_i$, where $\hat{X}$ is a set of maximizers of

$$\hat{V}(x, \mathbf{q}) = V(x, x, \mathbf{q}) = xP(\hat{Q}(x)),$$

which implies that $\hat{x}$ has to be a solution of

$$\frac{d\hat{V}(x, \hat{Q}(x))}{dx} = xP'(\hat{Q}(x))Q'(x) + P = 0. \tag{11}$$

∎

Lemma 1 *The equilibrium of the game $\hat{\Gamma}$ is unique if $P''' \leq 0$.*

Proof. of Lemma 1. There exists a unique equilibrium of the game $\hat{\Gamma}$, when $P''' \leq 0$, because in this case the derivative of equation (11) is negative for any x:

$$xP'(\hat{Q}(x))\frac{d^2\hat{Q}(x)}{dx^2} + xP''(\hat{Q}(x))\left[\frac{d\hat{Q}(x)}{dx}\right]^2 + 2P'(\hat{Q}(x))\frac{d\hat{Q}(x)}{dx} < 0,$$

where $\frac{d^2\hat{Q}(x)}{dx^2} < 0$:

$$\frac{d^2\hat{Q}(x)}{dx^2} = \frac{2i}{(1 - x)^3 A'(\hat{Q}(x))} + \frac{iA'''(\hat{Q}(x))}{(1 - x)^2 \left[A'(\hat{Q}(x))\right]^2} < 0, \tag{12}$$

$$A''(Q) = \frac{1}{N}P'''(Q) + \left[1 - \frac{1}{N}\right]\frac{1}{Q}\left(P''(Q) - \frac{2}{Q}\left[P'(Q) - \frac{P(Q)}{Q}\right]\right) \leq 0, \tag{13}$$

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because

$$P''(Q) - \frac{2}{Q}[P'(Q) - \frac{P(Q)}{Q}] \leq 0 \quad \text{if} \ \ P'''(Q) \leq 0.$$

∎

Equilibrium Dependence on the Number of Investors

Schwartz (2000) established that in both games, Γ and $\hat{\Gamma}$, the equilibrium asset size increases in N:

$$\frac{dQ^*}{dN} > 0 \quad \text{and} \quad \frac{d\hat{Q}}{dN} > 0. \tag{14}$$

Lemma 2 *Let* $P'''(Q) \leq 0$, *then* $\frac{d\hat{x}}{dN} > 0$.

[**Lemma 2.**][how equilibrium shares vary with the number of investors]. First, we prove our Lemma for the game $\hat{\Gamma}$. From Lemma 1, its equilibrium is unique, from equations (10) and (11) we have in equilibrium:

$$\frac{x}{i(1-x)^2} = -\frac{P}{P'} \times A'. \tag{15}$$

Differentiate equation (15) with respect to N and use equation (14) provides:

$$\frac{(1+x)}{i(1-x)^3} \times \frac{dx}{dN} = -\underbrace{\left[\frac{P'}{P'} - \frac{PP''}{[P']^2}\right]}_{[+]} \underbrace{\frac{d\hat{Q}}{dN}}_{[+]} \times \underbrace{A'}_{[-]} - \frac{P}{P'} \underbrace{A''}_{[-]} \frac{d\hat{Q}}{dN},$$

where the right hand side is positive from equations (9) and (13). Thus, the left hand side is positive too, which gives us:

$$\frac{d\hat{x}}{dN} > 0,$$

and Lemma 2. ∎

Lemma 3 *Let* $P'''(Q) \leq 0$, *and* $B''' \geq 0$ *then* $\frac{dy^*}{dN} > 0$.

Proof. We notice that the system of equations *(5) - (6) has a unique solution* $(x(y), Q(y))$ for any $y \in [0,1]$, and it is *continuous, twice continuously differentiable,* and $Q'(y) < 0$:

$$Q'(y) = \frac{i}{(1-y)^2 A'(Q)} < 0,$$

[[We are using y as a state variable here and below.]] Notice, that the function $x(y)$ is just an inverse of the function $y(x)$, which used in Schwartz (2000) proofs. Let V_y be defined as

$$V_y = V(x(y), y, Q(y)),$$

where $(x(y), Q(y))$ is a solution of the system of equations *(5) - (6)*. It is easy to see that the principal's problem is equivalent to the maximization of the function V_y

$$V_y = \max_{y \in [0,1]} yP(Q(y)) - B(y - x(y)). \tag{16}$$

Differentiation of equations *(5) - (6) and the use of the implicit function theorem provides that* $Q'(y)$ *and* $Q''(y)$ *are negative (similar to the game* $\hat{\Gamma}$*), and* $x'(y) \geq 1$ and $x'(y) \geq 0$

$$Q'(y) = \frac{i}{(1-y)^2 A'(Q(y))} < 0, \tag{17}$$

$$Q''(y) = \frac{2i}{(1-y)^3 A'(Q(y))} + \frac{iA'''(Q(y))}{(1-y)^2 [A'(Q(y))]^2} < 0 \tag{18}$$

$$\frac{P'}{B''}Q'(y) = 1 - x'(y) \quad \text{or} \quad x'(y) = 1 - \frac{P'}{B''}Q'(y) > 1, \tag{19}$$

$$x''(y) = -\frac{P''}{B''}[Q'(y)]^2 - \frac{P'}{B''}Q''(y) + \frac{P'B'''[1 - x'(y)]}{[B'']^2}Q'(y) > 0. \tag{20}$$

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Differentiate equation (16) and use equations *(17) and (19)* to show that in equilibrium of the game Γ:

$$yP'Q'(y) + Px'(y) = 0,$$

which differentiation with respect to N provides:

$$\frac{d}{dN}\left[\frac{y}{i(1-y)^2}\right] = -x'(y)\frac{d}{dN}\left[\frac{P}{P'}A'\right]_{[-]} - \frac{P}{P'}A'x''(y)\frac{dy}{dN}$$

$$\frac{dy^*}{dN}\left[\frac{(1+y)}{\underset{[+]}{i(1-y)^3}} + \frac{P}{P'}\underset{[+]}{x''(y)}\right] = -\underset{[+]}{x'(y)} \times \left\{\left[\frac{P'}{P'} - \frac{PP''}{\underset{[+]}{[P']^2}}\right]\frac{dQ^*}{dN} \times \underset{[-]}{A'} - \frac{P}{P'}\underset{[-]}{A''}\frac{dQ^*}{dN}\right\},$$

which proves that $\frac{dy^*}{dN} > 0$, and Lemma is proven. $\blacksquare$

Lemma 4 *If in the game Γ $\frac{dy^*}{dN} < 0$, then $\frac{dx^*}{dN} \leq 0$.*

Proof. (Lemma 4). From equation (5) and (14)

$$\frac{dy^*}{dN} > \frac{dx^*}{dN},$$

thus, $\frac{dx^*}{dN} < 0$ if $\frac{dy^*}{dN} < 0$. $\blacksquare$

Differentiate equation (5) with respect to x and use equation (8) to show

$$P'Q'(x) = B'' \times (y'(x) - 1) < 0,$$

$$y'(x) = 1 + \frac{P'}{B''}Q'(x),$$

$$0 < y'(x) < 1. \tag{21}$$

[[For $\forall x \in [0, 1]$, there exists a unique equilibrium the subgame of the game Γ that starts at x, and equilibrium schedules $Q(x, y) = Q^*(x)$ and $y(Q, x) = y^*(x)$ are continuous and convex in x (need to recheck the sign of second derivative of $y^*(x)$): $\frac{dQ^*(x)}{dx} < 0$, $\frac{d^2Q^*(x)}{dx^2} < 0$, $\frac{dy^*(x)}{dx} > 0$, $\frac{d^2y^*(x)}{dx^2} > 0$.]]

Proof of Theorem 2

We call an equilibrium in which player payoff is his maximum equilibrium payoff this player's *preferred* equilibrium, and denote $E^P \subset E$ and $E^I \subset E$ the sets of principal's and investors' preferred equilibria (PPE and IPE), where E is the set of game G equilibria. Clearly, in any equilibrium, player payoffs (V^e, Π^e) belong to the intervals bounded by their preferred equilibria payoffs:

$$V^e \in [V^I, V^R] \text{ and } \Pi^e \in [\Pi^R, \Pi^I].$$

Let an equilibrium in which the principal's surplus share equals s be called an s-surplus split equilibrium:

$$s = \frac{V^e}{S^e}$$

where $S^e = V^e + \Pi^e$ is player surplus in the outcome $\mathbf{o}^e$, and $e \in E$. In PPE and IPE, the s-surplus split is respectively the highest and the lowest. Let

$$S(x, Q, y) = V(x, Q, y) + \Pi(x, Q, y)$$

denote player surplus in the outcome $\mathbf{o}$ with actions (x, Q, y). Let $U(x, Q)$ be maximum principal's payoff (which is his payoff from deviation) for the *ex ante* action x and investment Q:

$$U(x, Q) = u(x, Q)P(Q) - B(u(x, Q) - x),$$

where $u(x, Q)$ is equal to

$$u(x, Q) = \arg\max_{z}[zP(Q) - B(z - x)],$$

thus,

$$U(x, Q) = V(x, Q, u(x, Q)).$$

Let $\Delta_u(x,Q)$ and $H(x,Q)$ denote

$$\Delta_u(x,Q) = u(x,Q) - x \quad \text{and} \quad H(x,Q) = \Delta_u(Q)P(Q) - B(\Delta_u(Q)).$$

Let $T(Q)$ denote the total gain from investment Q:

$$T(Q) = P(Q) - iQ = V(x,Q,y) + \Pi(x,Q,y) + B(y-x).$$

Then, T and S are related:

$$S(x,Q,y) = T(Q) - B(y-x).$$

Principal's incentive constraint (PIC) holds strictly for efficiency when player equilibrium surplus is below Pareto frontier.

Summary of Theorem 2 Proof

From continuity and compactness of the action space and quasi-convexity of player payoffs, there always exists an equilibrium of the game $G(\delta)$, see, for example, Fudenberg and Tirole, (1991).

Let $\hat{\delta}^P$ define a discount factor at which PIC for the outcome with actions $(\hat{y}, \hat{Q}, \hat{y})$ holds strictly.

Next, we prove the second part of Theorem 2: equilibrium uniqueness for $\delta \in [0, \hat{\delta})$, and stationary negotiation equilibrium.

We prove that if $V^\odot < \hat{V}$, in IPE and PPE $Q^\odot = \hat{Q}(y^\odot)$, and show that these PPE and IPE are unique and coincide. We also notice that when PPE and IPE are unique and coincide, the equilibrium of the game is unique. Then, we show that the assumptions of $V^\odot < \hat{V}$ is equivalent to $\delta \in [0, \hat{\delta})$. Lastly, we demonstrate that in the game G, for $\delta \in [0, \hat{\delta})$ PPE is a *reneging* equilibrium, and prove its stationarity.

Step 1: There exists a unique $u(x,Q)$ for any fixed $x \in [0,1]$ and $Q \in (0, \hat{Q}_\infty(0))$, and when u is interior (ie. $u \in (0,1)$) the functions U and u are increasing in Q for any fixed x, and increasing in x from any fixed Q:

$$\left.\frac{\partial u(x,Q)}{\partial Q}\right|_{x=const} > 0 \quad \text{and} \quad \left.\frac{\partial U(x,Q)}{\partial Q}\right|_{x=const} > 0,$$

$$\left.\frac{\partial u(x,Q)}{\partial x}\right|_{Q=const} > 0 \quad \text{and} \quad \left.\frac{\partial U(x,Q)}{\partial x}\right|_{Q=const} > 0,$$

and when $u(x,Q)$ is interior ($u(x,Q) \in (0,1)$) and the functions Δ_u and H depend only on Q (not on x).

Proof. From Theorem 1 there exists a unique $u(x,Q)$, which can be found as the solution of equation

$$P(Q) - B'(u-x) = 0. \tag{22}$$

Keep Q and x fixed, and differentiate equation (22) with respect to u. Since the derivative is negative, there exists a unique $u(x,Q)$ for any fixed Q and x. For any interior $u(x,Q)$ (i.e. $u(x,Q) \in (0,1)$), the expression $\Delta_u(x,Q) = u(x,Q) - x$ does not depend on x – it is fully determined by Q, and from equation (22) the function $\Delta_u(Q)$ increases in Q

$$\frac{d\Delta_u(Q)}{dQ} > 0.$$

Thus, for any fixed x,

$$\left.\frac{\partial u(x,Q)}{\partial Q}\right|_{x=const} > 0 \quad \text{and} \quad \left.\frac{\partial U(x,Q)}{\partial Q}\right|_{x=const} = uP' > 0,$$

and for any fixed Q:

$$\frac{du(x,Q)}{dx} = \frac{d[x + \Delta_u]}{dx} = 1 > 0,$$

and

$$\left.\frac{\partial U(x,Q)}{\partial x}\right|_{Q=const} = \frac{\partial u}{\partial x}[P - B'(\Delta_u)] + B'(\Delta_u) = P(Q) > 0,$$

and we have:

$$H(x,Q) = U(x,Q) - xP(Q) = (u-x)P(Q) - B(u-x) = \Delta_u P(Q) - B(\Delta_u),$$

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Thus, H is the function of Q (but not of x), and Step 1 is proven. ∎

Step 2. Equilibrium investment $Q^{\odot}$ in the game G in bounded by Q^{ρ}, where Q^{ρ} is Pareto optimal investment, [which means that Q^{ρ} is a solution of equation:

$$T'(Q) = 0 \Leftrightarrow P'(Q^{\rho}) = i.]$$

Proof. Let $\mathbf{o}^{\odot}$ be an equilibrium outcome with actions $(x^{\odot}, Q^{\odot}, y^{\odot})$, where $Q^{\odot} > Q^{\rho}$. Then

$$\left. \frac{dT(Q)}{dQ} \right|_{Q=Q^{\odot}} < 0,$$

and there exists $\tilde{y} > y^{\odot}$ such that

$$\Pi(\cdot, Q^{\rho}, \tilde{y}) = \Pi^{\odot}.$$

Let $x^{\odot} = y^{\odot}$, then:

$$T^{\rho} - T^{\odot} = \tilde{y} P(Q^{\rho}) - y^{\odot} P(Q^{\odot}) > 0,$$

and, thus, in the outcome $\tilde{\mathbf{o}}$ with actions $(\tilde{y}, Q^{\rho}, \tilde{y})$ we have:

$$V(\tilde{y}, Q^{\rho}, \tilde{y}) > V^{\odot},$$

From Step 1, the outcome $\tilde{\mathbf{o}}$ is sustainable and it is Pareto dominates $\mathbf{o}^{\odot}$. Thus, in a non-negotiation equilibrium $(x^{\odot} = y^{\odot})$ we have: $Q^{\odot} \leq Q^{\rho}$. Next, let $x^{\odot} < y^{\odot}$ and consider a deviation to the outcome $\tilde{\mathbf{o}}$ with actions $(\tilde{x}, Q^{\rho}, \tilde{y})$, where $\tilde{x} = \tilde{y} - [y^{\odot} - x^{\odot}]$. From Step 1, PIC holds in $\tilde{\mathbf{o}}$, thus, $\tilde{\mathbf{o}}$ is Pareto superior to $\mathbf{o}^{\odot}$, because

$$V(\tilde{x}, Q^{\rho}, \tilde{y}) = T^{\rho} - T^{\odot} + V^{\odot} > V^{\odot}$$
$$\Pi(\cdot, Q^{\rho}, \tilde{y}) = \Pi^{\odot}$$

Therefore, in a reneging equilibrium $Q^{\odot} \leq Q^{\rho}$, and Step 2 is proven. ∎

Step 3. The maximum equilibrium investment is $Q^{P\odot}$ (PPE investment):

$$\max_e Q^{\odot e} = Q^{P\odot}.$$

Proof. Let the outcome $\mathbf{o}^{1\odot}$ with actions $(x^{1\odot}, Q^{1\odot}, y^{1\odot})$ be PPE and the outcome $\mathbf{o}^{2\odot}$ with actions $(x^{2\odot}, Q^{2\odot}, y^{2\odot})$ be an equilibrium of the game G in which $Q^{1\odot} < Q^{2\odot}$. From $V^{1\odot} \geq V^{2\odot}$

$$\left[\Delta^{1\odot} P^{1\odot} - B^{1\odot}\right] - \left[\Delta^{2\odot} P^{2\odot} - B^{2\odot}\right] \geq x^{2\odot} P^{2\odot} - x^{1\odot} P^{1\odot}. \tag{23}$$

Then, from PIC: $U^{1\odot} - U^{2\odot} > 0$ and $U^{1\odot} - U^{2\odot} - V^{1\odot} - V^{2\odot} \geq 0$:

$$H^{1\odot} - H^{2\odot} + \left[x^{1\odot} P^{1\odot} - x^{2\odot} P^{2\odot}\right] > 0, \tag{24}$$
$$x^{1\odot} P^{1\odot} - x^{2\odot} P^{2\odot} > 0,$$

and

$$H^{1\odot} - H^{2\odot} - \left\{\left[\Delta^{1\odot} P^{1\odot} - B^{1\odot}\right] - \left[\Delta^{2\odot} P^{2\odot} - B^{2\odot}\right]\right\} > 0. \tag{25}$$

Since $Q^{1\odot} < Q^{2\odot}$ we have $H^{1\odot} - H^{2\odot} < 0$, because the function H increases in Q. Thus, equation (25) holds only if

$$\left[\Delta^{1\odot} P^{1\odot} - B^{1\odot}\right] - \left[\Delta^{2\odot} P^{2\odot} - B^{2\odot}\right] < 0.$$

Combined with equation (23) it provides:

$$0 > \left[\Delta^{1\odot} P^{1\odot} - B^{1\odot}\right] - \left[\Delta^{2\odot} P^{2\odot} - B^{2\odot}\right] \geq x^{2\odot} P^{2\odot} - x^{1\odot} P^{1\odot},$$

which contradicts equation (24). Thus, $Q^{1\odot} \geq Q^{2\odot}$, and Step 3 is proven. ∎

Step 4. Lemma PPE. If $V^{P\odot} < \hat{V}$ PPE is a *reneging* equilibrium, and $Q^{P\odot} = \hat{Q}(y^{P\odot})$, where $\hat{Q}(y)$ is unique (from Theorem 1).

Proof. We consider two cases: *reneging* PPE and *non-reneging* PPE.

I. First, let PPE be a *reneging* equilibrium: an outcome $\mathbf{o}^\odot$ with actions $(x^\odot, Q^\odot, y^\odot)$,

in which $Q^{\odot} \neq \hat{Q}(y^{\odot})$, $x^{\odot} < y^{\odot}$. Let

$$Q^{P\odot} < \hat{Q}(y^{P\odot}).$$

Then, the principal's PPE payoff can be increased by switching to an outcome $\tilde{\mathbf{o}}$ with actions $(\tilde{x}, \tilde{Q}, \tilde{y})$, such that $\tilde{x} = x^{\odot} + \Delta x$, $\tilde{Q} = Q^{\odot}$, $\tilde{y} = y^{\odot} + \Delta y$, and $\Delta x > 0, \Delta y > 0$, PIC holds strictly in the outcome $\tilde{\mathbf{o}}$, and

$$Q^{\odot} = \hat{Q}(\tilde{y}),$$

because from Theorem 1, the function $\hat{Q}$ is continuous and decreasing in y. Then, from Step 1:

$$\tilde{U} - U^{\odot} = \Delta x P^{\odot} > 0,$$

and from the properties of the function U, and PICs for the outcomes $\mathbf{o}^{\odot}$ and $\tilde{\mathbf{o}}$: we have:

$$(1 - \delta)\left(\tilde{U} - U^{\odot}\right) = \tilde{V} - V^{\odot} > 0,$$

because PIC can be rewritten as

$$(1 - \delta)\Delta x P^{\odot} = \Delta y P^{\odot} + B'^{\odot}(\Delta x - \Delta y) > 0,$$

where

$$B^{\odot} > \tilde{B}(y^{\odot} + \Delta y - x^{\odot} - \Delta x) \approx B^{\odot} - B'^{\odot}\left(\Delta x - \Delta y\right),$$

and $\Delta x > \Delta y$. [[where $U^{\odot} = U(x^{\odot}, Q^{\odot})$, and $\tilde{U} = U(\tilde{x} > x^{\odot}, \tilde{Q} = Q^{\odot})$.]] From Theorem 1, the function $\hat{Q}$ decreases with y, thus, there exists $\tilde{y} > y^{\odot}$, such that

$$Q^{\odot} = \hat{Q}(\tilde{y}),$$

which provides that the outcome $\tilde{\mathbf{o}}$ is compatible with investor incentives. [[Obvi-

ously, ceteris paribus, the principal's payoff increases with Q, and profit maximization of the short-lived investors provides $Q = \hat{Q}(y^\odot)$]].

Therefore, we have proven that $Q^\odot \geq \hat{Q}(y^\odot)$. Let

$$Q^\odot > \hat{Q}(y^\odot), \tag{26}$$

from which we have:

$$\Pi^\odot < \hat{\Pi}(y^\odot),$$

where $\Pi^\odot = \Pi(x^\odot, Q^\odot, y^\odot) = \Pi(\cdot, Q^\odot, y^\odot)$ and $\hat{\Pi}(y^\odot) = \Pi(\cdot, \hat{Q}(y^\odot), y^\odot)$. Since the function $\hat{\Pi}$ decreases in y, there exists $\breve{y} > y^{P\odot}$ such that

$$\Pi^\odot = \hat{\Pi}(\breve{y}) < \hat{\Pi}(y^\odot),$$

where $\hat{Q}(\breve{y}) < \hat{Q}(y^\odot) < Q^\odot$ and $\hat{\Pi}(\breve{y}) = \Pi(\cdot, \hat{Q}(\breve{y}), \breve{y})$. Clearly, for any $y \in (y^\odot, \breve{y})$ there exists $\tilde{Q} \in (\hat{Q}(\breve{y}), Q^\odot)$ such that

$$\Pi(\cdot, \tilde{Q}, \tilde{y}) = \Pi^\odot, \tag{27}$$

where

$$Q^\odot - \tilde{Q} = \Delta Q > 0, \tag{28}$$
$$\tilde{y} - y^\odot = \Delta y > 0, \tag{29}$$

with $\Delta Q \in (0, Q^\odot - \hat{Q}(\breve{y}))$ and $\Delta y \in (0, \breve{y} - y^\odot)$. For any Q^1, y^1 and Q^2, y^2, with $Q^1 < Q^2$

$$\Pi(\cdot, Q^1, y^1) = \Pi(\cdot, Q^2, y^2) = \Pi^\odot,$$

we have $y^1 > y^2$. From Step 1, there exists $\tilde{x} = x^\odot + \Delta x$, where $\Delta x > 0$ such that

$$U(\tilde{x}, \tilde{Q}) = U(x^\odot, Q^\odot), \tag{30}$$

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and $\Delta y - \Delta x < 0$. To prove Main Lemma, we need to show that there exists $\tilde{V} = V(\tilde{x}, \tilde{Q}, \tilde{y})$ such that

$$\Delta V = \tilde{V} - V^{\odot} > 0, \tag{31}$$

where $V^{\odot} = V(x^{\odot}, Q^{\odot}, y^{\odot})$. From the above, we have:

$$\Delta V = -[P'^{\odot} - i]\Delta Q - B'^{\odot}[\Delta y - \Delta x], \tag{32}$$

where $B'^{\odot} = B'(y^{\odot} - x^{\odot})$ and $P'^{\odot} = P'(Q^{\odot})$. From equation (27):

$$T(Q^{\odot}) - T(Q^{\odot} - \Delta Q) = -\Delta y[P^{\odot} - P'^{\odot}\Delta_Q] + y^{\odot}P'^{\odot}\Delta Q, \tag{33}$$
$$[(1 - y^{\odot})P'^{\odot} - i]\Delta Q = -\Delta y[P^{\odot} - P'^{\odot}\Delta Q],$$

because Taylor approximation provides:

$$T(Q^{\odot}) - T(Q^{\odot} - \Delta Q) = [P'^{\odot} - i]\Delta Q.$$

From PIC and properties of the functions V and U, in PPE we have:

$$\left.\frac{dV(x^{\odot}, Q^{\odot}, y^{\odot})}{dx}\right|_{x=x^{\odot}} = 0,$$

$$\left.\frac{dy}{dx}[P^{\odot} - B'^{\odot}] + y^{\odot}P'^{\odot}\frac{dQ}{dx} + B'^{\odot}\right|_{x=x^{\odot}} = 0,$$

and for efficiency, PIC binds strictly, providing:

$$\left.\frac{dU(x^{\odot}, Q^{\odot})}{dx}\right|_{x=x^{\odot}} = 0 \Rightarrow u^{\odot}P'^{\odot}\frac{dQ}{dx} + P^{\odot}\right|_{x=x^{\odot}} = 0,$$

or

$$P'^{\odot}\frac{dQ}{dx} = \left.\frac{P^{\odot}}{u^{\odot}}\right|_{x=x^{\odot}}.$$

From the above presented equations and $P^{\odot} - B'^{\odot} > 0$ we have:

$$\frac{B'^{\odot}}{P^{\odot}} < \frac{y^{\odot}}{u^{\odot}}, \qquad (34)$$

where $u^{\odot} = u(x^{\odot}, Q^{\odot})$. From equation (30) we have:

$$\frac{\Delta Q}{\Delta x} = \frac{P^{\odot}}{u^{\odot} P'^{\odot}} \bigg|_{x=x^{\odot}}. \qquad (35)$$

Next, substitute equation (33) in (32):

$$\Delta V = \{-\Delta_y + \Delta_x - \Delta_x\}[P^{\odot} - P'^{\odot}\Delta Q] + (y^{\odot} + \Delta y)P'^{\odot}\Delta Q - B'^{\odot}[\Delta y - \Delta x],$$
$$\Delta V = (P - B'^{\odot})[\Delta_y - \Delta_x] - \Delta_x P^{\odot} + (y^{\odot} + \Delta y)P'^{\odot}\Delta_Q,$$

divide the last equation by $\Delta x P^{\odot}$, and use equation (35) to show:

$$\frac{\Delta V}{\Delta x P^{\odot}} = \left[1 - \frac{B'^{\odot}}{P^{\odot}}\right]\left[1 - \frac{\Delta y}{\Delta x}\right] - 1 + \frac{(y^{\odot} + \Delta y)}{u^{\odot}},$$

and

$$\frac{\Delta V}{\Delta x P^{\odot}} = \left[\frac{y^{\odot}}{u^{\odot}} - \frac{B'^{\odot}}{P^{\odot}}\right] + \frac{B'^{\odot}}{P^{\odot}}\frac{\Delta y}{\Delta x} + \frac{\Delta y}{u^{\odot}} \geq 0$$

because from equation (34)

$$\frac{y^{\odot}}{u^{\odot}} - \frac{B'^{\odot}}{P^{\odot}} > 0.$$

Therefore, equation (32) holds, and Lemma PPE is proven if PPE is a reneging equilibrium. (Investor incentive constraint holds in the outcome $\tilde{\mathbf{o}}$, because the same punishment as for the outcome $\mathbf{o}^{\odot}$ deters investor deviation.) ∎

II. Next, we assume a non-*reneging* PPE – an outcome $\mathbf{o}^{\odot}$ with actions $(y^{\odot}, Q^{\odot}, y^{\odot})$ and $Q^{\odot} \neq \hat{Q}(y^{\odot})$, and show that it cannot be an equilibrium.

IIa. Let $Q^{\odot} < \hat{Q}(y^{\odot})$, and consider an outcome $\tilde{\mathbf{o}}$ with actions $(\tilde{y}, \tilde{Q} = \hat{Q}(\tilde{y}) = Q^{\odot}, \tilde{y})$. By construction, $\tilde{y} > y$, thus, PIC holds in $\tilde{\mathbf{o}}$ and the principal's payoff is higher than in the outcome $\mathbf{o}^{\odot}$. The outcome $\tilde{\mathbf{o}}$ is compatible with investor incen-

tives, thus, it cannot be the PPE because principal's payoff is higher in in $\tilde{\mathbf{o}}$, which is sustainable.

IIb. Let $Q^\odot > \hat{Q}(y^\odot)$. Then, an increase in $Q^\odot$ is Pareto improving.

Since in this Lemma we assume $V^{P\odot} < \hat{V}$, an outcome $(\tilde{x} = \tilde{y}, \hat{Q}, \tilde{y})$ is not sustainable (if $\tilde{x} \geq \hat{x}$, $\tilde{V}(\tilde{x} = \tilde{y}, \hat{Q}, \tilde{y}) \geq \hat{V}$, which conflicts $V^{P\odot} < \hat{V}$ and if $\tilde{x} < \hat{x}$, $\hat{Q} < \hat{Q}(y^\odot)$, which conflicts our assumption $Q^\odot > \hat{Q}(y^\odot)$). Thus, $Q^\odot < \hat{Q}$.

Then, we have $y^\odot > \hat{x}$ (because $Q^\odot > \hat{Q}(y^\odot)$, $Q^\odot < \hat{Q}$ and the function $\hat{Q}$ is decreasing.)

To prove IIb, let an outcome $\mathbf{o}^{1\odot}$ with actions $(x^\odot \leq y^\odot, Q^{1\odot}, y^\odot)$ be PPE for $V^{1\odot} < \hat{V}$. Then, there exists $\epsilon > 0$, such that for any x in the ϵ-vicinity of $x^\odot = y^\odot$,(i.e., $x \in X_\epsilon$, where X_ϵ denotes ϵ-vicinity of $x^\odot$, $(x^\odot - \epsilon, y^\odot)$), there exists a unique $Q^\odot(x) > Q^{1\odot}$, such that

$$U^\odot(x) = U(x, Q^\odot(x)) = U^{1\odot}.$$

And there exists a unique $y^\odot(x) < x^\odot$, such that for any outcome $\mathbf{o}^\odot(x)$ with actions $(x, Q^\odot(x), y^\odot(x))$ PIC holds strictly. From our construction:

$$V^{1\odot} = V^\odot(x) = V(x, Q^\odot(x), y^\odot(x)).$$

Then, for x converging to $x^\odot$ we have:

$$\left.\frac{dS^\odot(x)}{dQ}\right|_{x \longrightarrow x^\odot} = \left.\frac{dT^\odot(x)}{dQ} - B'(\Delta^\odot(x))\frac{d[\Delta^\odot(x)]}{dQ}\right|,$$

where

$$\Delta^\odot(x) = y^\odot(x) - x,$$

and $\left.\frac{d\Delta^\odot(x)}{dQ}\right|_{x \longrightarrow x^\odot}$ is, clearly, finite, and $B'(\Delta^\odot(x))|_{x \longrightarrow x^\odot} = 0$, while $\left.\frac{dT^\odot(x)}{dQ}\right|_{x \longrightarrow x^\odot} > 0$. Therefore,

$$\left.\frac{dS^\odot(x)}{dQ}\right|_{x \longrightarrow x^\odot} > 0,$$

from which there exists a *reneging* outcome $\mathbf{o}^{\odot}(x)$, with $x < x^{\odot}$, Pareto superior to $\mathbf{o}^{1\odot} = \mathbf{o}^{\odot}(x^{\odot})$. Thus, $\mathbf{o}^{1\odot}$ cannot be an equilibrium, and II is proven.

Thus, for any $V^{P\odot} < \hat{V}$ PPE is a *reneging* equilibrium, in which case we have shown in I. that $Q^{P\odot} = \hat{Q}(y^{P\odot})$, and Lemma PPE is proven.

Thus, if $V^{P\odot} < \hat{V}$ we have $Q^{P\odot} = \hat{Q}(y^{P\odot})$ and PPE is a *reneging* equilibrium. [[Below, in Proposition 1 we derive the properties of the PPE equilibrium, using our result of $Q^{\odot} = \hat{Q}(y^{\odot})$, for the properties of the functions $\hat{Q}$ and $\hat{V}$ see Theorem .]]

Step 5. Lemma IPE. If $V^{I\odot} < \hat{V}$ in any IPE $Q^{I\odot} = \hat{Q}(y^{I\odot})$.

Proof of Step 5. The result is obvious for $N = 1$ and tedious for $N > 1$.

First, we notice that the lowest discount factor $\hat{\delta}^{P}$ at which the principal's PPE payoff reaches $\hat{V}$ increases in N:

$$\frac{d\hat{\delta}^{P}}{dN} > 0,$$

because the asset value $\hat{P}$ increases in N. Thus, we have:

$$\hat{\delta}^{P}_{N2} > \hat{\delta}^{P}_{N1},$$

if $N2 > N1$.

Proof of Step 5 From Step 4, at any $\delta < \hat{\delta}^{P}$ we have $V^{P\odot} = yP(Q(y(x)) - B(y - x) < \hat{V}$

We let $\hat{\delta}^{P}$ denote the discount factor at which $V^{P\odot} = \hat{V}$, and we will show that if $V^{I\odot} < \hat{V}$, in any IPE $Q^{I\odot} = \hat{Q}(y^{I\odot})$.

Proof of Lemma IPE, (Theorem 1)

I. We show that if $\Pi^{I\odot} < \Pi^{P\odot}$, we have $Q^{I\odot} < \hat{Q}(y^{I\odot})$.

Proof of I: Assume the reverse. Let $Q^{I\odot} > \hat{Q}(y^{I\odot})$, then from Steps 3 and 4:

$$\hat{Q}(y^{I\odot}) < Q^{I\odot} \leq Q^{P\odot} = \hat{Q}(y^{P\odot}),$$

and since the function $\hat{Q}$ decreases in y:

$$y^{I\odot} > y^{P\odot}.$$

Then, investor profit is higher in PPE than in IPE:

$$\Pi^{I\odot} < \hat{\Pi}^{I\odot} < \hat{\Pi}^{P\odot} = \Pi^{P\odot},$$

where

$$\Pi^{I\odot} = \Pi(x^{I\odot}, Q^{I\odot}, y^{I\odot}), \quad \hat{\Pi}^{I\odot} = \Pi(\cdot, \hat{Q}(y^{I\odot}), y^{I\odot}),$$
$$\Pi^{P\odot} = \Pi(x^{P\odot}, Q^{P\odot}, y^{P\odot}), \quad \hat{\Pi}^{P\odot} = \Pi(\cdot, \hat{Q}(y^{P\odot}), y^{P\odot}),$$

which contradicts the definition of PPE and IPE. Thus, we have shown that $Q^{I\odot} \leq \hat{Q}(y^{I\odot})$.

II. If at some $\delta < \hat{\delta}^P$ we have: $\Pi^{I\odot} < \Pi^{P\odot}$ (which is equivalent to $Q^{I\odot} < \hat{Q}(y^{I\odot})$), then at $\delta < \hat{\delta}^P$ we have $\Pi^{I\odot} < \Pi^{P\odot}$ (which is $Q^{I\odot} < \hat{Q}(y^{I\odot})$).

Proof of II: Indeed, assume that at some $\delta^1 < \hat{\delta}^P$ the outcomes $\mathbf{o}^{I\odot}$ and $\mathbf{o}^{P\odot}$ with actions $(x^{I1\odot}, Q^{I1\odot}, y^{I1\odot})$ and $(x^{P1\odot}, Q^{P1\odot}, y^{P1\odot})$ are nonidentical:

$$Q^{I1\odot} < Q^{P1\odot} \quad \text{and} \quad \Pi^{I1\odot} > \Pi^{P1\odot}, V^{I1\odot} < V^{P1\odot},$$

$$\pi^1 = \Pi^{I1\odot} - \Pi^{P1\odot} > 0.$$

Then, clearly, there exists IPE, in which $\Pi^{I2\odot} > \hat{\Pi}$ at $\delta = \hat{\delta}^P$

$$\hat{\pi} = \Pi^{I2\odot} - \hat{\Pi} > 0.$$

To see that let $V^{I2} = V^{I1\odot}$, with $V^{I2} = \hat{y} P^{I2\odot} < \hat{V} = \hat{y}\hat{P}$. Since $P^{I2} < \hat{P}$, PIC holds for the outcome $(\hat{y}, P^{I2}, \hat{y})$ and

$$\Pi^{I2\odot} \geq \Pi^{I2} > \hat{\Pi}.$$

III. If $\delta = \hat{\delta}^P$, we have $Q^{I\odot} = \hat{Q}(y^{I\odot})$.

Proof of III: From I $Q^{I\odot} \leq \hat{Q}(y^{I\odot})$. Thus, to prove III, we have to show only that $Q^{I\odot} < \hat{Q}(y^{I\odot})$ cannot be IPE. Let $N = 1$, then, increase in $Q^{I\odot}$ is, clearly,

Pareto improving. Thus, III holds for $N = 1$:

$$Q_1^{I\odot} = \hat{Q}(y^{I\odot}),$$

and combined with results of Steps 3 and 4, it provides that for $N = 1$ the equilibrium of the game G is unique:

$$Q_1^{P\odot} = Q_1^{I\odot} = \hat{Q}(y^{I\odot}) \quad \text{and} \quad S_1^{P\odot} = S_1^{I\odot}.$$

Next, let $N = 2$. Then, from comparing PICs in the games with $N = 1$ and $N = 2$ we have:

From I and II, to prove Lemma IPE it is sufficient to show that $Q^{I\odot} = Q^{P\odot}$ at $\delta = \hat{\delta}^P$, which was shown in III, thus, Lemma IPE is proven.

Step 6. When $V^{P\odot} < \hat{V}$ the IPE and PPE are unique.

Proof of Step 6: Assume there exist two PPE (or IPE), outcomes $\mathbf{o}^{1\odot}$ and $\mathbf{o}^{2\odot}$ with actions $(x^{1\odot}, Q^{1\odot}, y^{1\odot})$ and $(x^{2\odot}, Q^{2\odot}, y^{2\odot})$. We denote:

$$U^{e\odot} = U(x^{e\odot}, P^{e\odot}), \quad V^{e\odot} = V(x^{e\odot}, P^{e\odot}, y^{e\odot}) \quad \text{and} \quad \Pi^{e\odot} = \Pi(x^{e\odot}, P^{e\odot}, y^{e\odot})$$
$$\Delta^{e\odot} = y^{e\odot} - x^{e\odot} \quad \text{and} \quad B^{e\odot} = B(\Delta^{e\odot}), \quad \text{where } e = 1, 2.$$

Let $Q^{1\odot} < Q^{2\odot}$, then in both cases, PPE (or IPE) we have from Step 4 (or 5) than when $V^{P\odot} < \hat{V}$ $Q^{P\odot} = \hat{Q}(y^{P\odot})$ (or $Q^{I\odot} = \hat{Q}(y^{I\odot})$), in which case

$$\Pi^{1\odot} < \Pi^{2\odot},$$

which contradicts $\Pi^{1\odot} = \Pi^{2\odot}$. Therefore, when $V^{P\odot} < \hat{V}$ in all PPE (or IPE) the asset value is the same:

$$Q^{1\odot} = Q^{2\odot},$$

and, thus,

$$y^{1\odot} = y^{2\odot},$$

and from $V^{1\odot} = V^{2\odot}$:

$$y^{1\odot}P^{1\odot} - y^{2\odot}P^{2\odot} = B^{1\odot} - B^{2\odot} = 0,$$

we have:

$$x^{1\odot} = x^{2\odot}.$$

Thus, the outcomes $\mathbf{o}^{1\odot}$ and $\mathbf{o}^{2\odot}$ are identical, providing that PPE (and IPE) are unique, and Step 6 is proven.

Step 7. When $V^{P\odot} < \hat{V}$ PPE and IPE coincide, the equilibrium of the game G is unique.

To Prove Step 7, we show that (I) when $V^{P\odot} < \hat{V}$ PPE and IPE coincide and (II) when PPE and IPE coincide in the game G, its equilibrium is unique.

I. From Steps 4 and 5 we have: $Q^{P\odot} = \hat{Q}(y^{P\odot})$ and $Q^{I\odot} = \hat{Q}(y^{I\odot})$. Then:

$$y^{I\odot} \geq y^{P\odot} \text{ and } Q^{P\odot} \leq Q^{I\odot}, \tag{36}$$

because from Theorem 1 investor profit and the function $\hat{Q}$ are decreasing in y. But equation (36) is agreeable with Step 3 (equilibrium asset value is the highest in PPE) only when $Q^{P\odot} = Q^{I\odot}$, in which case:

$$y^{I\odot} = y^{P\odot} \quad \text{and} \quad x^{I\odot} = x^{P\odot},$$

and PPE and IPE coincide. Next, from the definition of PPE and IPE,

$$V^{P\odot} = \max_e V^{e\odot} \quad \text{and} \quad \Pi^{I\odot} = \max_e \Pi^{e\odot}.$$

Thus, the equilibrium of the game G is unique when $V^{P\odot} < \hat{V}$.

II: When PPE and IPE coincide, we have:

$$V^{P\odot} = V^{I\odot} = V^{\odot} \text{ and } \Pi^{P\odot} = \Pi^{I\odot} = \Pi^{\odot}.$$

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From definition of player preferred equilibria:

$$V^{e\odot} \in [V^{I\odot}, V^{R\odot}] \text{ and } \Pi^{e\odot} \in [\Pi^{R\odot}, \Pi^{I\odot}],$$

and we have:

$$V^{e\odot} \equiv V^{\odot} \text{ and } \Pi^{e\odot} \equiv \Pi^{\odot}.$$

Thus, an equilibrium of the game G is unique, because from Step 6, PPE (and IPE) are unique when $V^{P\odot} < \hat{V}$, and Step 7 is proven.

Step 8. The assumption of $V^{\odot} < \hat{V}$ and $\delta < \hat{\delta}_G$ are equivalent.

From the above presented we have:

$$V^{\odot} < \hat{V} \Leftrightarrow \Pi^{\odot} < \hat{\Pi},$$

Thus, if $V^{\odot} < \hat{V}$ we have $S^{\odot} < \hat{S}$. Thus, when $V^{\odot} < \hat{V}$ we have $\delta < \hat{\delta}$, and when $V^{\odot} = \hat{V}$ we have $\Pi^{\odot} = \hat{\Pi}$. Thus, the commitment outcome is reached when $\delta = \hat{\delta}$, but not at any lower δ.

Step 9. For any $\delta \in [0, \hat{\delta})$, the equilibrium of the game G is negotiation equilibrium.

Follows from Lemma PPE (where we prove that PPE is a negotiation equilibrium for $V^{\odot} < \hat{V}$), and Step 8 ($V^{\odot} < \hat{V}$ is equivalent to $\delta < \hat{\delta}_G$).

Step 10. From equilibrium uniqueness, stationary of the equilibrium is automatic.

Proof of the stationarity of the equilibrium: Assume the reverse: let player equilibrium payoffs differ in some periods a and b. Then for efficiency:

$$\Pi^a \leq \Pi^b \quad V^a \geq V^b.$$

It is clear from above that when

$$\Pi^a = \Pi^b \quad V^a = V^b,$$

player actions in periods a and b are identical. So, assume that

$$\Pi^a < \Pi^b \quad V^a > V^b, \tag{37}$$

then from PIC

$$V^a - V^b = U^a - U^b,$$

in which case if $V^a < V^b$ we have $\Pi^a < \Pi^b$, which contradicts equation 37, and Step 10 is proven, and Theorem 2 is proven as we have shown existence, uniqueness and stationarity of the equilibrium.

Proof of Proposition 1

Let the outcome $\mathbf{o}^{P\odot}$ with actions $(x^\odot, Q^\odot, y^\odot)$ denote PPE outcome in the game $G(\delta)$.

Lemma 1. There exists $\epsilon > 0$, such that for any x in the ϵ-vicinity of $x^\odot$,(i.e., $x \in X_\epsilon$, where X_ϵ denotes ϵ-vicinity of $x^\odot$, $(x^\odot - \epsilon, x^\odot + \epsilon)$), there exists unique $y^\odot(x)$ and $Q^\odot(y) = \hat{Q}(y^\odot(x))$, such that for the outcome $\mathbf{o}^\odot(x)$, with actions $(x, Q^\odot(x), y^\odot(x))$ PIC holds strictly.

Proof of Lemma 1. The existence and uniqueness of $\mathbf{o}^\odot(y)$ follows from Theorems 1 and 2. Let $\mathbf{O}^\odot(X_\epsilon)$ denote the set of outcomes $\mathbf{o}^\odot(x)$. Then the PPE outcome $\mathbf{o}^{P\odot} = \mathbf{o}^\odot(x^\odot) \in \mathbf{O}^\odot(X_\epsilon)$. From the principal's maximization in PPE, differentiation of $V^\odot(x)$ with respect to x at $x = x^\odot$ provides:

$$\left. \frac{dV^\odot(x)}{dx} \right|_{x=x^\odot} = 0,$$

$$y^\odot P'^\odot \frac{dQ^\odot(x)}{dx} + P - P + \frac{dy^\odot(x)}{dx} P^\odot - B'^\odot \left[\frac{dy^\odot(x)}{dx} - 1 \right] \Bigg|_{x=x^\odot} = 0,$$

$$yP'(Q) \frac{dQ^\odot(x)}{dx} + P + \left(P^\odot - B'^\odot \right) \left[\frac{dy^\odot(x)}{dx} - 1 \right] \Bigg|_{x=x^\odot} = 0,$$

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and since PIC holds strictly for efficiency we have at $x = x^\odot$

$$\left.\frac{dU^\odot(x)}{dx}\right|_{x=x^\otimes} = 0,$$

$$\left. u^\odot(x)P'^\odot\frac{dQ^\odot(x)}{dx} + P \right|_{x=x^\otimes} = 0 \Leftrightarrow -\frac{P}{u} = P'\frac{dQ^\odot(x)}{dx},$$

thus, we have:

$$P^\odot\left(1 - \frac{y^\odot(x)}{u^\odot(x)}\right) = -\left(P^\odot - B'^\odot\right)\left.\left[\frac{dy^\odot(x)}{dx} - 1\right]\right|_{x=x^\odot} = 0,$$

which provides

$$0 < \frac{dy^\odot(x)}{dx} < 1.$$

Then

$$\frac{dy^*(x)}{dx} < \frac{dy^\odot(x)}{dx} < \frac{du^\odot(x)}{dx}$$
$$Q^* < Q^\odot < \hat{Q},$$

from which:

$$x^* < x^\odot < \hat{y} < y^\odot < y^*.$$

By construction, in our equilibrium

$$\frac{dV^\odot}{d\delta} > 0, \tag{38}$$

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From Theorem 1, for any $y > \hat{y}$ the derivative $\left.\frac{d^2\hat{V}}{dy^2}\right|_{y>\hat{y}} < 0$ we have:

$$\left.\frac{dyP^{\otimes}(y)}{dy}\right|_{y=y^{\otimes 1}} > \left.\frac{dyP^{\otimes}(y)}{dy}\right|_{y=y^{\otimes 2}} \Leftrightarrow y^{\otimes 1} < y^{\otimes 2},$$

$$\left|\left.\frac{dyP^{\otimes}(y)}{dy}\right|_{y=y^{\otimes 1}}\right| < \left|\left.\frac{dyP^{\otimes}(y)}{dy}\right|_{y=y^{\otimes 2}}\right| \Leftrightarrow y^{\otimes 1} < y^{\otimes 2},$$

and using equation (**??**)

$$\left|B'\left.\frac{d\Delta^{\otimes}(y)}{dy}\right|_{y=y^{\otimes 1}}\right| < \left|B'\left.\frac{d\Delta^{\otimes}(y)}{dy}\right|_{y=y^{\otimes 2}}\right|$$

From Theorem 2 we know that $Q^{\odot} = \hat{Q}(y^{\odot})$, and $Q^{\odot} < Q^{\rho}$. Let y^{ρ} denote the principal's share such that: $\hat{Q}(y^{\rho}) = Q^{\rho}$.

When $y^{\odot} > \hat{y}$ (and, thus, $Q^{\odot} < \hat{Q}$) we have:

$$\frac{dT^{\odot}}{d\delta} - \frac{dy^{\odot}P^{\odot}}{d\delta} > 0 \Leftrightarrow \frac{dQ^{\odot}}{d\delta} > 0 \quad \text{and} \quad \frac{dy^{\odot}}{d\delta} < 0. \tag{39}$$

Lastly, we show that

$$\frac{dB^{\odot}}{d\delta} > 0. \tag{40}$$

To prove this, we notice that from Theorem 2, at $\delta = \hat{\delta}$ we have:

$$Q^{\odot}(\hat{\delta}) = \hat{Q} \quad \text{and} \quad x^{\odot}(\hat{\delta}) = y^{\odot}(\hat{\delta}) = \hat{y},$$

and from equation (39) at any $\delta \in (0, \hat{\delta})$ we have

$$Q^{\odot}(\delta) < \hat{Q} \quad \text{and} \quad y^{\odot}(\delta) > \hat{y}.$$

From equations (**??**), (**??**) and (**??**):

$$\frac{dV^{\odot}(x(\delta),\delta)}{d\delta} = \left.\frac{\partial V^{\odot}(x)}{\partial x}\right|_{\delta=const} \times \frac{dx}{d\delta} + \left.\frac{\partial V^{\odot}(x)}{\partial \delta}\right|_{x=const} > 0,$$

$$\left.\frac{\partial V^{\odot}(x,\delta)}{\partial \delta}\right|_{x=const} > 0,$$

$$\left.\frac{dy^{\odot}}{d\delta}\right|_{x=const} \times P^{\odot} + y\left.\frac{dP^{\odot}}{d\delta}\right|_{x=const} - B'^{\odot}\left.\frac{dy^{\odot}}{d\delta}\right|_{x=const} > 0,$$

Where $\left.\frac{dQ^{\odot}(x)}{d\delta}\right|_{x=const} > 0$, and, thus: $\left.\frac{\partial y^{\odot}(x)}{\partial \delta}\right|_{x=const} < 0$

$$\frac{1}{1-\delta}y^{\odot}\frac{dP^{\odot}}{d\delta} + \frac{1}{1-\delta}B'^{\odot}\left[\frac{dy^{\odot}}{d\delta} - \frac{dx^{\odot}}{d\delta}\right]$$

$$\frac{\partial}{\partial \delta}\left[U^{\odot}(x) - \frac{1}{1-\delta}V^{\odot}(x)\right]\Bigg|_{x=const} + \left\{\frac{\partial U^{\odot}(x)}{\partial x}\frac{dx}{d\delta} - \frac{1}{1-\delta}\frac{\partial V^{\odot}(x)}{\partial x}\frac{dx}{d\delta}\right\} = \frac{1}{[1-\delta]^2}\left[V^{\odot} - V^*\right] > 0,$$

where the term in curly brackets is equal to zero due to equation (**??**), which provides:

$$\frac{du^{\odot}}{d\delta}P^{\odot} + u^{\odot}\frac{dP^{\odot}}{d\delta} - P^{\odot}\left[\frac{du^{\odot}}{d\delta} - \frac{dx^{\odot}}{d\delta}\right] - \frac{1}{1-\delta}P^{\odot}\frac{dy^{\odot}}{d\delta} - \frac{1}{1-\delta}y^{\odot}\frac{dP^{\odot}}{d\delta} + \frac{1}{1-\delta}B'^{\odot}\left[\frac{dy^{\odot}}{d\delta} - \frac{dx^{\odot}}{d\delta}\right] > 0,$$
$$\tag{41}$$

$$\frac{dx^{\odot}}{d\delta}P^{\odot} + \left[u^{\odot} - \frac{1}{1-\delta}y^{\odot}\right]\frac{dP^{\odot}}{d\delta} - \frac{1}{1-\delta}\left[P^{\odot} - B'^{\odot}\right]\frac{dy^{\odot}}{d\delta} - \frac{1}{1-\delta}B'^{\odot}\frac{dx^{\odot}}{d\delta} > 0,$$

where $\frac{du^{\odot}}{d\delta} > 0$ because .... Equation (41) holds only if the last term is negative, that is

$$\frac{dx^{\odot}}{d\delta} > 0, \tag{42}$$

because all its other terms on the right hand side are positive, and we have shown

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that the left hand side is negative. From equations (38) and (42) we have:

$$\frac{dB(y^{\odot} - x^{\odot})}{d\delta} < 0, \tag{43}$$

To finish the proof notice that from our definition of $\hat{\delta}$

$$\lim_{\delta \to \hat{\delta}} x^{\odot}(\delta) = \lim_{\delta \to \hat{\delta}} y^{\odot}(\delta) = \hat{x},$$

and at $\delta = \hat{\delta}$ we have:

$$x^{\odot}(\hat{\delta}) = y^{\odot}(\hat{\delta}) = \hat{x} = \hat{y},$$

from equation (43) and the continuity of principal's payoff in $B(y - x)$, thus, Proposition 1 is proven.

Proof of Proposition 2

Let $\delta < 1$, and let parameters and variables have the subscripts Gl and G indicating the considered game. When possible, we drop this subscript to simplify notation. From PICs in the games G and Gl:

$$\hat{U} - \hat{V} = \frac{\hat{\delta}_G}{1 - \hat{\delta}_G}\left[\hat{V} - V^*\right] \quad \text{and} \quad \hat{U} - \hat{V} = \frac{\hat{\delta}_{Gl}}{1 - \hat{\delta}_{Gl}}\hat{V},$$

where $\hat{V} = V(\hat{x}, \hat{Q})$ and $\hat{U} = U(\hat{x}, \hat{Q})$, we have:

$$\hat{\delta}_G = \frac{\hat{U} - \hat{V}}{\hat{U} - V^*} < \hat{\delta}_{Gl} = \frac{\hat{U} - \hat{V}}{\hat{U}} \quad \text{and} \quad \Delta\delta = \frac{1 - \frac{\hat{V}}{\hat{U}}}{\frac{\hat{U}}{V^*} - 1} > 0,$$

which provides $\Delta\delta = \hat{\delta}_G - \hat{\delta}_{Gl} > 0$, (and $\Delta\delta$ decreases in $\frac{\hat{V}}{\hat{U}}$ and $\frac{\hat{U}}{V^*}$). From Theorems 2 and 3 for any $\delta < \hat{\delta}_{Gl}$ the equilibrium in each game is unique. Thus, to prove Proposition 2, it is sufficient to show that

$$x_G^{\odot} < x_{Gl}^{\odot} < \hat{x} = \hat{y} < y_{Gl}^{\odot} < y_G^{\odot} \quad \text{and} \quad Q_{Gl}^{\odot} > Q_G^{\odot} \tag{44}$$

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holds in some equilibrium. Consider PPE; for any $\delta < \hat{\delta}_{Gl}$ PICs in the games Gl and G hold strictly

$$U_{Gl}^{\odot} - V_{Gl}^{\odot} = \frac{\delta}{1-\delta} V_{Gl}^{\odot} \quad \text{and} \quad U_{G}^{\odot} - V_{G}^{\odot} = \frac{\delta}{1-\delta} V_{G}^{\odot} - \frac{\delta}{1-\delta} V^{*},$$

where $U^{\odot} = U(x^{\odot}, Q^{\odot})$, $V^{\odot} = V(x^{\odot}, Q^{\odot}, y^{\odot})$. Thus, we have:

$$V_{Gl}^{\odot} > V_{G}^{\odot},$$

because if the principal employs $x_{Gl}^{\odot}$ as his *ex ante* action in the game Gl, his payoff in the game Gl would be higher than $V_{G}^{\odot}$. Since for $\delta \in (0, \hat{\delta}_{Gl})$ investor profit increases as principal's payoffs increases, equilibrium profit is higher in the game Gl than G. We subtract PICs of the games G and Gl from each other:

$$U_{Gl}^{\odot} - U_{G}^{\odot} = \frac{1}{1-\delta} \left[V_{Gl}^{\odot} - V_{G}^{\odot} \right] + \frac{\delta}{1-\delta} V^{*} > 0 \quad \Rightarrow \quad U_{Gl}^{\odot} > U_{G}^{\odot}.$$

From Proposition 1 we have in the game G:

$$\left. \frac{dU_{G}^{\odot}}{dx} \right|_{x=x_{G}^{\odot}} = 0 \quad \text{and} \quad \left. \frac{dV_{G}^{\odot}}{dx} \right|_{x=x_{G}^{\odot}} = 0.$$

Similarly, in the game Gl:

$$\left. \frac{dU_{Gl}^{\odot}}{dx} \right|_{x=x_{Gl}^{\odot}} = 0 \quad \text{and} \quad \left. \frac{dV_{Gl}^{\odot}}{dx} \right|_{x=x_{Gl}^{\odot}} = 0.$$

[[[See my MFN paper for the complete derivation] Next, evaluate $\left. \frac{dU_{Gl}(x_{G}^{\odot}, \tilde{Q})}{dx} \right|_{x=x_{G}^{\odot}}$ and show that

$$\left. \frac{dU_{Gl}(x_{G}^{\odot}, \tilde{Q})}{dx} \right|_{x=x_{G}^{\odot}} > 0,$$

where from PICs we have $Q_G^{\odot} < \tilde{Q} < \hat{Q}$ and $y < \tilde{y} < y_G^{\odot}$. Thus,

$$x_G^{\odot} < x_{Gl}^{\odot}$$

because $\frac{d^2 U_{Gl}}{dx^2} < 0$. Similarly, we evaluate $\frac{dV_{Gl}}{dx}$ at $\tilde{x}$ such that $Q_{Gl}(\tilde{x}) = Q_G^{\odot}$ and $y_{Gl}(\tilde{x}) = y_G^{\odot}$. In this case $\tilde{x} > x_G^{\odot}$ and

$$\left. \frac{dV_{Gl}}{dx} \right|_{x=\tilde{x}} < \left. \frac{dV_G^{\odot}}{dx} \right|_{x=x_G^{\odot}} = 0.$$

Thus, $x_{Gl}^{\odot} < \tilde{x}$, which provides:

$$y_{Gl}^{\odot} < y_G^{\odot},$$

and from the properties of the function $\hat{Q}$:

$$Q_{Gl}^{\odot} > Q_G^{\odot},$$

because from Theorems 2 and 3 $Q_{Gl}^{\odot} = \hat{Q}(y_{Gl}^{\odot})$ and $Q_G^{\odot} = \hat{Q}(y_G^{\odot})$, and Proposition 2 is proven.]]

Proof of Theorem 4

Let Dl denote the game D, in which investors can use zero investment to punish the principal's deviation. The game Dl can be seen as the game D, in which investors are long-lived, but constrained from strategic actions, i.e., each period they maximize their per period profit. We use this definition of the game Gl to simplify the proofs[12]. Clearly, equilibria of the games D and Dl are similar, as the games are almost identical.

Step 1. In the game D action spaces of all players are continuous and compact, and player payoffs are quasi-convex. Thus, equilibrium exists.

Step 2. Let $\bar{V}$ be the maximum principal's payoff sustainable in the game G without negotiations, i.e., the maximum principal's payoff sustainable in outcome

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[12]It appears that our results hold even if investors can be strategic.

(x, Q, y) with $x = y$. Then:

$$\bar{V} < V^{\odot}. \tag{45}$$

Proof of Step 2: Let $\bar{V} \geq V^{\odot}$; then, from PICs at $(x^{\odot}, Q^{\odot})$ and $(\bar{y}, \bar{Q})$ we have:

$$U(x^{\odot}, Q^{\odot}) \leq U(\bar{y}, \bar{Q}). \tag{46}$$

From the proof of Proposition 1 in PPE we have: $\left.\frac{dU(x,Q)}{dx}\right|_{x=x^{\odot}} = 0$, and its second derivative with respect to x is negative for any $x \in [0, 1]$. Thus, the function U is decreasing for $x > x^{\odot}$:

$$U(x^{\odot}, Q^{\odot}) > U(x, Q),$$

which contradicts equation (46). Therefore, equation (45) holds, and Step 2 is proven.

Step 3. The principal's respective minmax payoffs in the games G and D are $\frac{V_G^{pun}(u)}{1-\delta}$ and $\frac{V_D^{pun}(u)}{1-\delta}$, where u denotes the ownership share to which he deviated. Then, $V_D^{pun}(u)$ is decreasing in u, and is lower than V^*:

$$V_D^{pun}(u) \leq V_G^{pun} < V^* \text{ and } \frac{dV_D^{pun}(u)}{du} \leq 0.$$

Obviously, for any $u \in (0, 1]$ we have $V_G^{pun} = V^*$. Next, we notice that principal can reach $V_D^{pun}(u)$ in the game G by employing his actions from the game D. Thus, $V_D^{pun}(u) < V_G^{pun} = V^*$. The same logic provides that $\frac{dV_D^{pun}(u)}{du} \leq 0$, and Step 3 is proven.

Step 4. When δ is close to zero, equilibrium of the game D is a negotiation equilibrium.

Proof of Step 4: When $\delta \to 0$ the highest asset value sustainable in the games D, G, Dl and Gl without negotiations converges to zero in all these games

$$\lim_{\delta \to 0} \bar{P}_D = \bar{P}_G = \bar{P}_{Gl} = \bar{P}_{Dl} = \bar{P} = 0,$$

because when $\delta \to 0$ PICs in the games D, G, Dl and Gl converge to each other:

$$H(\bar{Q}_D) = \frac{1}{1 - \delta}\bar{V}_D^* - \frac{\delta}{(1 - \delta)}V_D^{pun}(\bar{u});$$

$$H(\bar{Q}_G) = \frac{1}{1 - \delta}\bar{V}_G - \frac{\delta}{(1 - \delta)}V^*;$$

$$H(\bar{Q}_{Dl}) = H(\bar{Q}_{Gl}) = \frac{1}{1 - \delta}\bar{V}_{Dl} = \frac{1}{1 - \delta}\bar{V}_{Gl},$$

and $\bar{P} = 0$ because in the game G we have $\lim_{\delta \to 0} \bar{P}_G = 0$ (when $\delta \to 0$ the game G converges to the game Γ). Thus, principal's non-negotiation payoff sustainable in the games D, G, Dl and Gl converges to zero when $\delta \to 0$:

$$\lim_{\delta \to 0} \bar{V}_D = \bar{V}_G = \bar{V}_{Dl} = \bar{V}_{Gl} = 0. \tag{47}$$

Next, we show that there exists a 2-*cycle* with principal's per period payoff $\tilde{V}$ sustainable in the game D, such that

$$\tilde{V}(1 + \delta) = V(y^2, Q^1, y^1) + \delta V(y^1, Q^2, y^2);$$

where $y^1 = x_G^{\odot}$, $Q^1 = Q^*(y^2)$, $y^2 = y_G^{\odot}$, $Q^2 = Q_G^{\odot} = Q_G^{\odot}(x_G^{\odot}) = \hat{Q}(y_G^{\odot})$, and subscript and superscript $_G^{\odot}$ denote the equilibrium values for the game G.

From equation (47) there exists some $\delta > 0$ such that

$$\tilde{V} - \bar{V}_G > \bar{V}_D - \bar{V}_G,$$

because

$$\lim_{\delta \to 0} V_G^{\odot} - \bar{V} = V^* - 0 > 0.$$

Thus, $\lim_{\delta \to 0} \tilde{V} \neq 0$, and principal's payoff in the game D is strictly higher when his share is adjusted down infinitely often than his payoff from any monotone increasing sequence y^t, and Step 4 is proven.

Step 5. Let δ be close to zero. Then, player equilibrium payoffs are higher in the game D than G.

From Step 4, when $\delta \to 0$ in the games D and Dl principal's ownership share is adjusted down in equilibrium infinitely often. Thus, equilibria of the games D and Dl are negotiation equilibria.

Using Proposition 2 and taking the limits of $\delta \to 0$, similar to the proof of Step 4, there exists a 2-*cycle*, $(y_{Dl}^{1\star}, Q^2, y_{Dl}^\star)$, $(y_{Dl}^{2\star}, Q^1, y_{Dl}^{1\star})$ such that $y_{Dl}^{1\star} \in (x_G^\odot, x_{Gl}^{\odot 2})$ and $y_{Dl}^{2\star} \in (y_{Gl}^\odot, y_G^\odot)$, and $Q^2 = \hat{Q}(y_{Dl}^{2\star}) > Q_{Gl}^\odot$, and Q^1 is a solution of equation:

$$U_{Dl}^{1\star} - V_{Dl}^{1\star} = H(Q^1) + \Delta_{Dl}^{2\star} P^1 = \frac{\delta}{(1-\delta)} V_{Dl}^\star,$$

where $U_{Dl}^{1\star} = U(y_{Dl}^{2\star}, Q_{Dl}^{1\star})$ and $V_{Dl}^{1\star} = V(y_{Dl}^{2\star}, Q_{Dl}^{1\star}, y_{Dl}^{1\star}) = y_{Dl}^{1\star} P_{Dl}^{1\star}$, and PIC in the game Dl holds for $(y_{Dl}^{2\star}, Q_{Dl}^{1\star}, y_{Dl}^{1\star})$

$$U_{Dl}^{2\star} - V_{Dl}^{2\star} = H(Q^2) - \left[\Delta_{Dl}^{2\star} P_{Dl}^{2\star} - B_{Dl}^{2\star}\right] = \frac{\delta}{(1-\delta)} V_{Dl}^\star,$$

where $U_{Dl}^{2\star} = U(y_{Dl}^{1\star}, Q_{Dl}^{2\star})$ and $V_{Dl}^{2\star} = V(y_{Dl}^{1\star}, Q_{Dl}^{2\star}, y_{Dl}^{2\star}) = y_{Dl}^{1\star} P_{Dl}^{2\star} + \left[\Delta_{Dl}^{2\star} P_{Dl}^{2\star} - B_{Dl}^{2\star}\right]$, $\Delta_{Dl}^{2\star} = y_{Dl}^{2\star} - x_{Dl}^{2\star}$. where

$$V_{Dl}^{2\star} + \delta V_{Dl}^{1\star} = (1+\delta) V_{Dl}^\star,$$

and from combining PICs in the game Gl 2-*cycle*, $(y_{Dl}^{1\star}, Q^2, y_{Dl}^\star)$, $(y_{Dl}^{2\star}, Q^1, y_{Dl}^{1\star})$:

$$U_{Dl}^{2\star} + \delta U_{Dl}^{1\star} - (1+\delta) V_{Dl}^\star = \frac{\delta(1+\delta)}{(1-\delta)} V_{Dl}^\star,$$

and subtracting the PIC in the game G:

$$U_{Dl}^{2\star} + \delta U_{Dl}^{1\star} - (1+\delta) U_G^\odot = \frac{(1+\delta)}{(1-\delta)} \{V_{Dl}^\star - V_G^\star\} + \frac{\delta(1+\delta)}{(1-\delta)} V^*, \qquad (48)$$

or taking the limit $\delta \to 0$ we have

$$U_{Dl}^{2\star} - U_G^\odot = V_{Dl}^\star - V_G^\star,$$

thus, $U_{Dl}^{2\star} - U_G^\odot > 0$ does not converge to zero as $\delta \to 0$, when $V_{Dl}^\star - V_G^\odot > 0$, which

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clearly is possible because

$$V_{Dl}^{2\star} - V_G^{\odot} > 0,$$

and $V_{Dl}^{2\star} - V_G^{\odot}$ does not converge to zero as $\delta \to 0$, because as $\delta \to 0$ we have $V_{Dl}^{2\star} \to V_{Gl}^{\odot}$, from which

$$V_{Dl}^{\star} - V_G^{\odot} > 0$$

follows immediately. Notice that

$$V_{Dl}^{\star} - V_{Gl}^{\odot} < 0,$$

because $V_{Dl}^{2\star} \le V_{Gl}^{\odot}$, and $V_{Dl}^{1\star} < V_{Gl}^{\odot}$. [[we use this to prove that $V_{Dl}^{\star} < V_{Gl}^{\odot}$]]

The same technic provides that in the game D as $\delta \to 0$:

$$V_D^{\star} - V_G^{\odot} > 0,$$

the notation in the proof for the game D is more cumbersome because of terms $\frac{\delta}{(1-\delta)} V_D^{pun}(u^t)$. When $\delta \to 0$ these terms, clearly, do not converge to $\frac{\delta}{(1-\delta)} V^*$, from which the result $V_D^{\star} - V_G^{\odot} > 0$ follows.

Step 6. The game D has a negotiation equilibrium for any $\delta < \hat{\delta}$, and $V_D^{\star} - V_G^{\odot} > 0$.

Proof of Step 6: We consider PPE, and show that for any δ there exists an outcome in the game D, Prove that Let the change in δ be have the same effect on player equilibrium actions in the games D and G:

$$\frac{dy_D^{1\star neg}}{d\delta} = \frac{dx_G^{\odot}}{d\delta} > 0, \frac{dy_D^{t\star neg}}{d\delta} = \frac{dy_G^{\odot}}{d\delta} < 0, \text{ and } \frac{dQ_D^{t\star neg}}{d\delta} = \frac{dQ_G^{\odot}}{d\delta} > 0.$$

Then, PICs in the game D hold. Thus, for any $\delta < \hat{\delta}$ there exists negotiation outcome with a T-*cycle* $(y_D^{1\star neg})$, and investments $(Q_D^{t\star neg})$ in which

$$V_D^{\star neg} - V_G^{\odot} > 0,$$

(and $y_D^{2\star} < y_G^\odot$ and $Q_D^{2\star} > Q_G^\odot$.) Thus, we have shown that the principal's PPE payoffs is higher than his payoff in the game G.

Next, we show that from Step 5 (the game D has negotiation equilibrium for δ close to zero) the game D has a negotiation equilibrium for any $\delta < \hat\delta$.

Assume the reverse. Let the game D have a negotiation equilibrium for some $\delta^{non} < \hat\delta$, with equilibrium actions $(y_D^{\star non}, Q_D^{\star non}, y_D^{\star non})$. Then, there exists some $\tilde\delta < \delta^{non}$ at which principal's maximum non-negotiation and negotiation payoffs are the same:

$$V_D^{\star non} = V_D^\star > V_G^\odot,$$

and subtract these outcome's PICs:

$$\sum_{t=1}^{T} \delta^{t-1} \left\{ U_D^{\star non} - U^{t\star} \right\} = 0 = \frac{\delta}{1-\delta} \left[\sum_{t=1}^{T} \delta^{t-1} V_D^{pun}(u^{t\star}) - V_D^{pun}(u^{\star non}) \sum_{t=1}^{T} \delta^{t-1} \right] > 0,$$

to get a contradiction. Thus, the equilibrium in the game D is negotiation equilibrium for any $\delta < \hat\delta$, and Step 6 is proven.

Step 7. The proof is of equilibrium uniqueness in the game D is similar to the proof of uniqueness in the game G. There exists unique negotiation equilibrium, and unique T-cycle in the game D for $\delta \to 0$

Analog of Lemma Main (from the proof of Theorem 2) holds in the game D for $\delta \to 0$ there exists a unique $Q^{t\star} = \hat{Q}(y^{t\star})$ for each $y^{t\star}$. In this case, the considerations can be limited to the two first periods of the T-*cycle*. Let $V_D^\star$ denote the principal's PPE payoff.

Step 8. The proof of uniqueness of negotiation equilibrium is the same as in Theorem 2. From Steps 7 and 8, there exists a unique negotiation PPE and thus, a unique negotiation equilibrium in the game D for any $\delta \in (0, \hat\delta_D)$, and we have

$$V_D^\star > V_G^\odot \quad \text{and} \quad \Pi_D^\star > \Pi_G^\odot.$$

Proof of Proposition 3

Properties of equilibrium T-cycles

Let $V^\star$ be average principal's equilibrium per period payoff in the equilibrium T-*cycle* of the game D:

$$\sum_{t=1}^{T} \delta^{t-1} V^{t\star} = V^\star \sum_{t=1}^{T} \delta^{t-1}.$$

Then, PIC in period t is:

$$U^{t\star} - V^{t\star} = \frac{\delta}{1-\delta} V^\star - \frac{\delta}{1-\delta} V^{pun}(u^{t\star}),$$

where $U^{t\star} = U^\star(y^{(t-1)\star}, Q^{t\star})$, $V^{t\star} = V(y^{(t-1)\star}, Q^{t\star}, y^{t\star})$, and $u^{t\star} = u(y^{(t-1)\star}, Q^{\star t})$.

1. For any $t1$ and $t2$ $(1 < t1 < t2)$ from the T-*cycle*:

$$y^{t1\star} < y^{t2\star} \Leftrightarrow Q^{t1\star} > Q^{t2\star} \quad \text{and} \quad u^{t1\star} < u^{t2\star}, \Delta^{t1\star} < \Delta^{t2\star} = y^{t2\star} - y^{(t2-1)\star}$$

where $\Delta^{t\star} = y^{t\star} - y^{(t-1)\star}$, and $Q^{t1\star} > Q^{t2\star}$ follows immediately from investor optimization. Subtract PICs in periods $t1$ and $t2$ from each other to get:

$$\left[U^{t1\star} - U^{t2\star} \right] - \left[V^{t1\star} - V^{t2\star} \right] = \frac{\delta}{1-\delta} \left[V^{pun}(u^{t2\star}) - V^{pun}(u^{t1\star}) \right] < 0,$$

and

$$0 < \left[U^{t1\star} - U^{t2\star} \right] < \left[V^{t1\star} - V^{t2\star} \right] \Rightarrow V^{t1\star} > V^{t2\star} \Leftrightarrow U^{t1\star} > U^{t2\star}.$$

Thus, principal's payoff decreases with t for any $t > 1$, and 1 is proven.

From 1, we the principal's payoff decreases with t.

2. Then, $V^{t\star} > V^\star$ for any $t > 1$. Assume there exists $T1 < T$, such that $V^{(T1-1)\star} > V^\star$ and $V^{T1\star} < V^\star$. Then, the principal's payoff from the T1-*cycle* is higher than from the equilibrium T-*cycle*, which is a contradiction, and 2 is proven.

From 1 and 2 we have:

$$\sum_{t=2}^{T} \delta^{t-1} \left[V^{t\star} - V^\star \right] = V^\star - V^{1\star} > 0,$$

and Proposition 3 is proven

Proof of Corollary 3

Proof. Consider PPE, in the game D with $N \to \infty$ and $\delta < \hat{\delta}_D$ (which is the same as $V_D^\star < \hat{V}$). Then, from Theorem 4 the game D has negotiation equilibrium. Let $(y^{t\star})$ denote equilibrium T-*cycle*, and $Q^{t\star}$ – equilibrium investments. Since investors are short-lived, their profit is at least zero, and for any $t > 1$ in the PPE we have for $N \to \infty$ we have:

$$\Pi(y^{t\star}, Q^{t\star}) = 0,$$
$$T^t - y^{t\star} P^{t\star} = iQ^{t\star},$$

where $Q^{t\star} = \hat{Q}(y^{t\star})$ and thus, $Q^{t\star}$ decrease with t for $t = 1, \dots, T$. Since $y^{1\star}$ is the smallest of $y^{t\star}$, investor profit at $y^{1\star}$ is zero only if $Q^{1\star}$ is the highest among $Q^{t\star}$, which contradicts Proposition 3, where we have shown that $Q^{1\star}$ is the smallest. Therefore,

$$\Pi(y^{1\star}, Q^{1\star}) > 0,$$

and since equilibrium T-*cycle* has a finite length of T, in the game D equilibrium profits of perfectly competitive investors are strictly positive: $\Pi(y^{1\star}, Q^{1\star}) \frac{\delta^T}{1-\delta^T} > 0$, and Corollary 3 is proven. ∎

Proof of Theorem 5 (unclean)

PIC in the game G is:

$$U(x^\odot, Q^\odot) - V^\odot = \frac{\delta}{1 - \delta} \left[V^\odot - V^* \right]. \tag{49}$$

Next, we construct a 2-*cycle* that in which principal's per period payoff equals $V^\odot$. We notice that there exists an outcome $(x_D^\odot, Q^\odot, y^\odot)$ sustainable in the game D such

that

$$U(x_D^\odot, Q^\odot) - V(x^{D\odot}, Q^\odot, y^\odot) = \frac{\delta}{1-\delta} \left[V(x_D^\odot, Q^\odot, y^\odot) - V^{pun}(u^\dagger) \right], \qquad (50)$$

in which case

$$U(x_D^\odot, Q^\odot) - U(x^\odot, Q^\odot) = (x_D^\odot - x^\odot) P(Q^\odot) = \frac{1}{1-\delta} \left[B(\Delta_D^\odot) - B(\Delta^\odot) \right] + \frac{\delta}{1-\delta} \left[V^* - V^D(u_D^\odot) \right],$$

and we have $x_D^\odot > x^\odot$ and $V^D(u) < V^*$ and is decreasing in u. If $x^{D\odot} \leq x^\odot$, the last equation does not hold: its left hand side is negative, while its right hand side is positive.

Thus, $V(x_D^\odot, Q^\odot, y^\odot) > V^\odot$ and

$$\Delta_D^\odot = y^\odot - x_D^\odot < \Delta^\odot = y^\odot - x^\odot.$$

Then, we show that there exists an outcome $(x^\dagger, Q^\dagger = \hat{Q}(y^\dagger), y^\dagger)$ sustainable in the game D such that

$$V^\dagger = V(x^\dagger, Q^\dagger, y^\dagger) \geq V^\odot,$$

where

$$\Delta^\dagger = y^\dagger - x^\dagger \geq \Delta^\odot. \qquad (51)$$

Clearly, there exists $Q^\dagger > Q^\odot$ such that $V^\dagger = V(x^\dagger, Q^\dagger, y^\dagger) = V^\odot$ sustainable in the game D:

$$y^\dagger P(Q^\dagger) - B(\Delta^\dagger) = y^\odot P(Q^\odot) - B(\Delta^\odot),$$

and if equation (51) does not hold

$$y^\dagger P(Q^\dagger) - y^\odot P(Q^\odot) = B(\Delta^\dagger) - B(\Delta^\odot) < 0,$$

or

$$y^\dagger P(Q^\dagger) < y^\odot P(Q^\odot). \tag{52}$$

From the proof of Theorem 1 there exists $\tilde{y}P(\tilde{Q})) = y^\odot P(Q^\odot)$ (where $\tilde{Q} = \hat{Q}(\tilde{y})$) such that:

$$\tilde{y} < \hat{y} < y^\odot \text{ and } P(\tilde{Q}) > P(Q^\odot),$$

and from equation (52) if

$$y^\dagger > \tilde{y}$$

we have $V(x^\dagger, Q^\dagger, y^\dagger) > V^\odot$, and $V(\tilde{x}, \tilde{Q}, \tilde{y}) > V^\odot$, or $\tilde{y} - \tilde{x} < \Delta^\odot$, in which case

$$U(\tilde{y} - \Delta^\odot, \tilde{Q}) < U(\tilde{x}, \tilde{Q}).$$

Thus, the outcome $(\tilde{y} - \Delta^\odot, \tilde{Q}, \tilde{y})$ is sustainable in the game D: PIC in the game D holds for this outcome, $V(\tilde{y} - \Delta^\odot, \tilde{Q}, \tilde{y}) = V^\odot$, and equation (51) holds. To summarize, we have shown that either the outcome $(\tilde{y} - \Delta^\odot, \tilde{Q}, \tilde{y})$ is sustainable in the game D, or $B(\Delta^\dagger) > B(\Delta^\odot)$. When $B(\Delta^\dagger) > B(\Delta^\odot)$, we have $y^\dagger P(Q^\dagger) > y^\odot P(Q^\odot)$, and there exists an outcome $(\tilde{y}^\dagger - \Delta^\odot, \tilde{Q}^\dagger, \tilde{y}^\dagger)$ sustainable in the game D, in which $V(\tilde{y}^\dagger - \Delta^\odot, \tilde{Q}^\dagger, \tilde{y}^\dagger) > V^\odot$.

From Proposition [] we have $V_{Dl}^\star < V_{Gl}^\odot$, and $V_D^\star < V_{Dl}^\star$ because the principal's payoff $V_D^\star$ is achievable in the game Dl if principal employs his actions from the game D in the game Dl. Thus, we have proven that $V_D^\star < V_{Dl}^\star$, and $V_{Gl}^\odot < V_{Dl}^\star$. Next, let δ be close to zero. Then, if principal employs his actions from the game Dl in the game D, his payoff is at least $V_G^\odot$, and principal's equilibrium payoff in the game D increases with δ faster?? than his payoff in the game G. Thus, 3 is proven for any $\delta < \hat{\delta}_{Dl}$.

From Proposition 2 and 1 we have:

$$x^* < x_G^\odot < x_{Gl}^\odot < \hat{x} = \hat{y} < y_{Gl}^\odot < y_G^\odot < y^* \quad \text{and} \quad \hat{Q} > Q_{Gl}^\odot > Q_G^\odot > Q^*.$$

From Proposition 2 maximum principal's payoff sustainable in the game D:

$$V_G^\odot < V_D < V_{Gl}^\odot,$$

which follows from comparison of PICs in the games G, D and Gl:

$$U_{Gl}^\odot - V_{Gl}^\odot = \frac{\delta}{1-\delta} V_{Gl}^\odot < U_G^\odot - V_G^\odot = \frac{\delta}{1-\delta} \left[V_G^\odot - V^* \right]$$

$$< U_D - V_D = \frac{\delta}{1-\delta} \left[V^\star - V_D^{pun}(u) \right]$$

$$< \frac{\delta}{1-\delta} V^\star < \frac{\delta}{1-\delta} V_{Gl}^\odot = U_{Gl}^\odot - V_{Gl}^\odot,$$

where $0 < V_D^{pun}(u) < V^*$. Thus, we have shown that $V_G^\odot < V_D^\star < V_{Gl}^\odot$.